Recent Developments in Productivity and Efficiency Analysis in the Agricultural Sector

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1 Introduction

Measures of productivity growth and economic efficiency play a central role in agricultural economics because they provide empirical benchmarks for performance differences across farms, regions, and time, and because they connect observed outcomes to technology, constraints, and incentives. This paper reviews methodological advances in agricultural productivity and efficiency analysis with an emphasis on contributions since 2010.

Our review focuses on economically relevant measures of performance that preserve a functional representation of technology. Consequently, we prioritize distance functions and surplus functions (directional distance functions) over alternative approaches because they offer two critical advantages that are essential for rigorous economic analysis. First, they provide complete functional characterizations of the production technology . This allows the technology set to be described mathematically, ensuring a direct link between physical production possibilities and the efficiency measure itself. Second, these functions support duality results that link technical efficiency to economic optimization. Under standard axioms, distance and surplus functions form dual pairs with the cost, revenue, and profit functions. This duality is indispensable for applied work as it allows researchers to recover information about technology from economic data (and vice versa) and to interpret efficiency scores in terms of lost profit or excess cost. By maintaining this focus, we ensure that the measurement of efficiency remains tethered to the fundamental economic principles of cost minimization and profit maximization.

The remainder of this paper is organized as follows. Section 2 establishes the basic concepts of production, reviewing the standard axioms and formally defining the distance and surplus functions alongside their dual economic relationships. Section 3 surveys the core empirical methods for measuring productivity and efficiency, including Data Envelopment Analysis (DEA) , Stochastic Frontier Analysis (SFA) , and developments in robust nonparametric estimation and uncertainty. Section 4 then synthesizes recent agricultural applications organized around five themes: (i) heterogeneity and technology gaps, (ii) dynamics, adjustment, and long-run productivity, (iii) environment, by-production, and circularity, (iv) spatial and institutional constraints, and (v) uncertainty management. Section 5 concludes.

2 Basic concepts in production

2.1 The technology and standard axioms

The production technology transfers a vector of N inputs, $x \in \mathbb{R}_+^N$, into a vector of M outputs, $y \in \mathbb{R}_+^M$. This is represented by the technology set,

$$T = \left\{(x, y) \in \mathbb{R}_+^N \times \mathbb{R}_+^M : x \text{ can produce } y\right\}$$

Standard theory requires that T satisfy several axioms (Koopmans, 1951; Shephard, 1970; Fare et al., 1994). These include:

- **Non-emptiness:** The technology set T is not empty.
- **Closedness:** T is a closed set. This property ensures well-defined boundaries of T that are included in T.
- **Boundedness:** For any given finite vector of inputs x, the set $Y(x) \equiv \{y : (x, y) \in T\}$ is bounded. This implies that infinite amounts of outputs cannot be produced from finite inputs.
- **Free disposability of inputs and outputs:** if $(x, y) \in T$ then $(x^o, y^o) \in T$ for $(x^o, -y^o) \geq (x, -y)$. Generalizes the traditional notions of non-negative marginal products and marginal cost.
- **Convexity:** T is a convex set: $(x^1, y^1) \in T$ and $(x^2, y^2) \in T \Rightarrow \lambda(x^1, y^1) + (1 - \lambda)(x^2, y^2) \in T$ for $0 < \lambda < 1$. Convexity ensures T does not exhibit increasing returns.

2.2 Distance functions and Surplus functions

For each feasible input vector $x \in \mathbb{R}_{+}^{N}$, the *output set* is defined:

$$Y(x) \equiv \{\, y \in \mathbb{R}_{+}^{M} : (x, y) \in T \,\},$$

and for each $y \in \mathbb{R}_{+}^{M}$, the *input set* is

$$X(y) \equiv \{\, x \in \mathbb{R}_{+}^{N} : (x, y) \in T \,\}.$$

Following Shephard (1970), the *output distance function* is defined:

$$D_o(x, y) \;\equiv\; \inf\{\, \theta > 0 : y \in \theta Y(x) \,\}.$$

Under the standard axioms, D_o is sublinear (positively homogeneous and convex) in y and $Y(x) = \{y : D_o(x, y) \leq 1\}$. The *input distance function* is defined:

$$D_i(x, y) \;\equiv\; \sup\{\, \rho > 0 : x \in \rho X(y) \,\}$$

Under the standard axioms, D_i is superlinear (positively homogeneous and concave) in x and $X(y) = \{x : D_i(x, y) \geq 1\}$.

$D_o(x, y)$ and $D_i(x, y)$ provide radial characterizations of $Y(x)$ and $X(y)$. The output distance function identifies how much $Y(x)$ needs to be radially "shrunk" to bring y to its frontier. The input distance function how much $X(y)$ needs to be radially expanded to make x a frontier point. Figure 1 illustrates the derivation ofthe input distance function for $x^o \in X(y)$. $X(y)$ is everything on or to the northeast of solid curve labelled $X(y)$ and $D_i(x^o, y)X(y)$ is everthing on or to the northeast of the dashed curve.

The *surplus (directional-distance function)* (Luenberger, 1992; Chambers et al., 1996, 1998), which projects (x, y) towards $T's$ along a specified path $g = (g_y, g_x)$, generalizes the

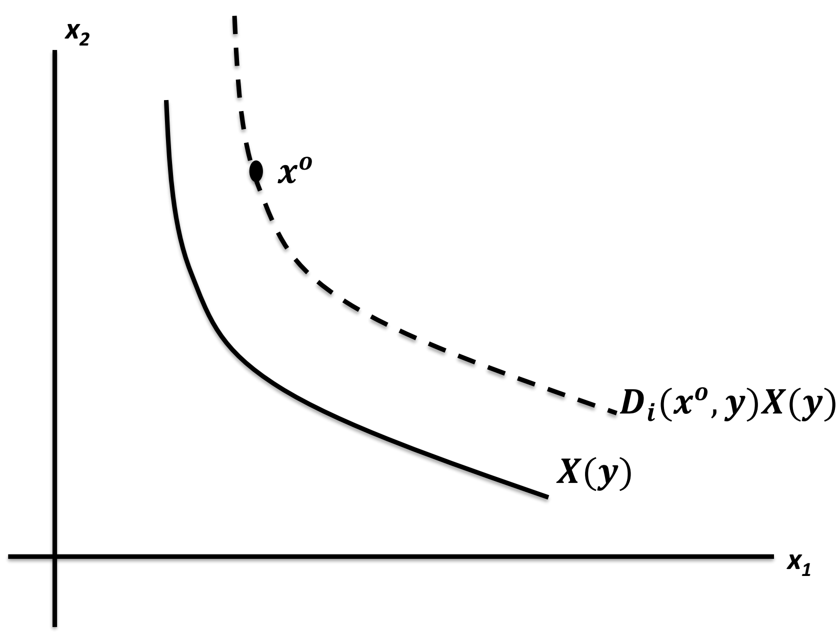

Figure 1: Input Distance Function

input and output distance functions.

$$D^g(x, y) \equiv \sup\{\, \beta \in \mathbb{R} : (\, x - \beta g_x,\ y + \beta g_y\,) \in T\,\}.$$

Altering g allows varying non-proportional expansions of selected outputs and contractions of selected inputs; the radial measures are recovered as special cases by taking g proportional to (y, x). Under the standard axioms D^g is concave in (x, y), satisfies the *Translation Property*:

$$D^g(x - \alpha g_x, y + \alpha g_y) = D^g(x, y) - \alpha, \tag{1}$$

and $T = \{(x, y) : D^g(x, y) \geq 0\}$.

2.3 Cost, Revenue, and Profit Functions

Under the standard axioms,the distance and surplus functions provide function characterizations of T that support theory and empirical practice. Each also forms a natural (well-known) dual pair with an indirect objective function (Shephard 1970; Luenberger 1992). The dual relations establish that either the primal or dual perspectives provide equivalent characterizations of technical relations.

The *cost function*, $c(w, y)$, gives the cheapest possible way to produce the output vector, y, at input prices, $w \in \mathbb{R}^N_+$. Under the standard assumptions, it forms the following dual pair with the input distance function:[1]

$$\begin{aligned} c(w, y) &\equiv \inf_x \{w'x : D_i(x, y) \geq 1\} \qquad (2) \\ D_i(x, y) &= \inf_w \{w'x : c(w, y) \geq 1\} \end{aligned}$$

The *revenue function*, $R(p, x)$, gives maximal feasible revenue for output prices, $p \in \mathbb{R}^M_+$,

[1] Debreu (1951)'s derivation of the *coefficient of resource utilization*, which is equivalent to a distance function, follows (2). Both (2) and (3) manifest the mathematical principles underlying the polar duality of the *gauge* and *support* functions for closed convex sets. See, for example, Rockafellar (1970) Theorem 14.5.

and input use x. It forms a dual pair with the output distance function.

$$\begin{aligned} R(p,x) &\equiv \sup_y \{p'y : D_o(x,y) \le 1\} \\ D_o(x,y) &= \sup_p \{p'y : R(p,x) \le 1\} \end{aligned} \tag{3}$$

The *profit function*, $\pi(p,w)$, gives maximal profit for prices (p,w). It forms a dual pair with the surplus function.

$$\begin{aligned} \pi(p,w) &\equiv \sup_{x,y} \{p'y - w'x : D^g(x,y) \ge 0\} \\ D^g(x,y) &= \inf_{p,w} \{\pi(p,w) - (p'y - w'x) : p'g_y + w'g_x = 1\} \end{aligned} \tag{4}$$

The essence of each of these dual relationships lies in the ability of distance and surplus functions to characterize T. In each case, that permits formulating the relevant economic optimization problem as a nonlinear program where the distance or surplus function characterizes the nonlinear constraint. We illustrate with surplus function. By the definition of $D^g(x,y)$, we know that $(x - D^g(x,y)g_x, y + D^g(x,y)g_y) \in T$ for all (x,y). Therefore it must be true that

$$\pi(p,w) \ge p'(y + D^g(x,y)g_y) - w'(x - D^g(x,y)g_x) \text{ for all } (x,y,w,p)$$

which implies that

$$\frac{\pi(p,w) - (p'y - w'x)}{p'g_y + w'g_x} \ge D^g(x,y) \text{ for all } (x,y,w,p).$$

Because this inequality holds for all (w,p), it surely holds when (w,p) are chosen to minimize the left-hand side. Our conditions on T suffice to ensure that the inequality holds as an equality at the minimizing (w,p).

Besides offering different perspectives on T, these dual relations characterize the economic principles underlying the usuage of distance and surplus function to attach *shadow (virtual)* prices to non-priced inputs or outputs.

2.4 Defining efficiency

Intuition suggests that an input-output bundle, (x, y), is *technically efficient* if it lies on the frontier of T. In terms of input or output distance functions, these are the input-output combinations where the respective distance function equals one. For surplus function, it's those (x, y) for which $D^g(x, y) = 0$.

An input-output bundle, (x, y), is usually defined to be *economically efficient* for prices (p, w) if $\pi(p, w) = p'y - w'x$.[2] The duality between T and $\pi(p, w)$ requires that if (x, y) is economically efficient, it must be technically efficient. Conversely, duality also requires that if T is nonempty closed convex, then technical efficiency ensures that there exist (w, p) for which (x, y) are technically efficient. So, it would seem that technical and economic efficiency are the same thing.

There is a catch, however. That intuition does not correspond with a common notion of technical efficiency that is attributed Koopmans (1957). Koopmans definition rests formally on the Paretian criterion attached to the traditional $\leq$ partial ordering of $\mathbb{R}^{M+N}$. It defines $(x, y) \in T$ as technically efficient if there exists no other $(x^o, y^o) \in T$ such that $(x^o, -y^o) \leq (x, y)$ with at least one inequality strict. Figure 2 illustrates a case where: A is on the frontier of the input set $X(y)$, A is economically efficient for input price constellations satisfying $w = (0, \hat{w}_2)$ with $w_2 > 0$, but A is inefficient in the Koopmans sense because usage of x_1 can be decreased without affecting the production of y. The *lacuna* between Koopman's efficiency definition and the above definition of economic efficiency is closed by limiting prices of inputs and outputs to be strictly positive.[3] In other words, all inputs and outputs are treated as economic "goods". While this notion of technical efficiency works well in many

[2] Cost efficiency for x and revenue efficiency for y are defined analogously.

[3] Technically, this requires that we restrict attention to prices falling in the relative interior of the effective domain of $\pi(p, w)$, which under free disposability of inputs and outputs contains only positive prices.

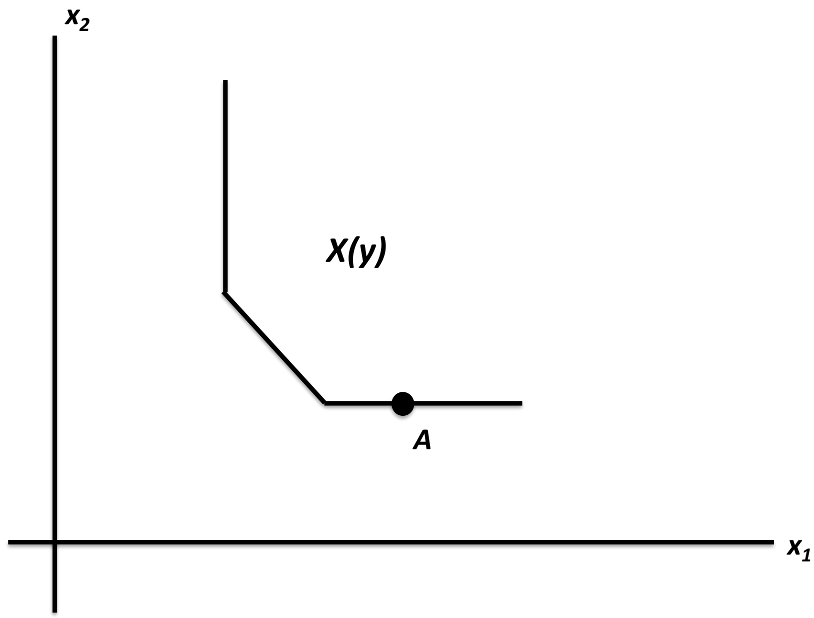

Figure 2: Koopmans Inefficiency

settings, it also fails in many practical setting. Agriculture, in particular, offers obvious and well-known examples that range from the existence of input congestion to the production of damaging environmental emissions and onto material-balance concerns. Each of these conflict with the traditional notion of freely disposable inputs and outputs upon which the Koopmans criterion rests.

Starting with Shephard's (1970) introduction of the notion of *weakly* disposable outputs, many attempts have been made to relax free disposability of inputs and outputs to accommodate its conflict with physical reality. Chambers (2026) develops a flexible framework for defining and measuring efficiency. Efficient outcomes are defined relative to a generalized partial ordering of $\mathbb{R}^{M+N}$, denoted $\preceq_C$, generated by a cone $C \subset \mathbb{R}^{\mathbb{M}+\mathbb{N}}$ that reflects specific disposability assumptions on inputs and outputs. (The traditional assumption of free disposability corresponds to a particular choice of C.) Under the assumption that T is a nonempty, closed, convex set that satisfies this generalized disposability property, Chambers (2026) develops a generalized inefficiency measure and conjugate duality for it that generalizes the duality results in (2), (3), and (4).

3 Empirical Methods for Measuring Productivity and Efficiency

3.1 Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a method for evaluating the relative efficiency of decision-making units (DMUs). Following the pioneering contributions of Farrell (1957), Afriat (1972), Charnes et al. (1978), and Banker et al. (1984), the DEA framework uses the standard axioms imposed on T and observed data to construct an empirical envelope of the observed data. That empirical envelope is then used to construct a conservative approximation to the "best-practice production frontier".

Given K observed DMUs (indexed $i = 1, ..., K$), each with N inputs (indexed $n = 1, ..., N$) and M outputs (indexed $m = 1, ..., M$), the canonical DEA framework constructs the approximation to T as the convex hull of observed activities using non-negative intensity variables z_i. The canonical conservative approximation to T is:

$$
\begin{aligned}
T_K = \Bigg\{ (x, y) : \quad & \sum_{k=1}^{K} z_k x_{kn} \leq x_n, \quad n = 1, \ldots, N; \\
& \sum_{k=1}^{K} z_k y_{km} \geq y_m, \quad m = 1, \ldots, M; \\
& \sum_{k=1}^{K} z_k = 1; \\
& z_k \geq 0, \quad k = 1, \ldots, K \Bigg\}
\end{aligned} \tag{5}
$$

An adaptation of this framework intended to accommodate inconsistencies of the free disposability axiom is to approximate T as the intersection of two sub-technologies: a "good-output" sub-technology that is freely disposable in y and an "emission" sub-technology in which b is *weakly* disposable, linked by accounting (material-balance) restrictions (Førsund, 2009; Murty et al., 2012; Førsund, 2018). In DEA, this yields a pair of intensity systems—one for producing y and one governing b—that jointly satisfy input constraints and the linking conditions. This framework disentangles production and abatement activities and enforces that reductions in b absorb resources or forgo good output, thereby operationalizing weak disposability without ad hoc transformations of the bads. While the exact specification depends on the empirical context (and available abatement variables), this adjustment to the model preserves relevant aspects of the standard model while still supporting a solid basis for environmental performance analysis (Førsund, 2009; Murty et al., 2012; Ray, 2022).

3.2 Stochastic Frontier Analysis (SFA)

Stochastic Frontier Analysis (SFA) offers a parametric alternative to DEA by modeling the frontier as a stochastic function. Unlike traditional regression methods that attribute all deviations from an estimated production relationship to statistical noise and assume full efficiency, SFA decomposes this deviation into statistical noise and inefficiency (Kumbhakar et al., 2022). This decomposition allows researchers to estimate the production technology and the extent of inefficiency for a sample or individual DMUs.

The benchmark Stochastic Frontier Model (SFM), independently proposed by Aigner et al. (1977) and Meeusen and van Den Broeck (1977), forms the foundation of SFA. The standard cross-sectional SFM is specified in its multiplicative form as:

$$y_i = f(x_i; \beta) \exp(v_i - u_i) \tag{6}$$

where y_i denotes the output of production unit i, x_i is a vector of inputs for unit i, and $f(\cdot)$ is a specific parametric production function (for example, Cobb-Douglas or Translog) with a vector of parameters β to be estimated. The term v_i represents statistical noise that accounts for measurement errors and other random shocks. The term u_i captures non-negative technical inefficiency ($u_i \geq 0$) representing the extent to which the unit i operates below the stochastic production frontier $f(x_i; \beta) \exp(v_i)$.

Taking natural logarithms of Equation (6) transforms this model into an additive one, which is convenient for estimation:

$$\ln(y_i) = \ln[f(x_i; \beta)] + v_i - u_i$$

Let $\ln[f(x_i; \beta)]$ be denoted as $f_{\ln}(x_i; \beta)$. The composed error term is $\epsilon_i = v_i - u_i$.To estimate the SFM and disentangle inefficiency u_i from noise v_i, distributional assumptions are imposed on these two error components (Aigner, Lovell, and Schmidt 1977; Meeusen and van Den Broeck 1977).

The application of SFA faces several econometric challenges. A key concern is input endogeneity that arises from input choices being correlated with unobserved shocks (v_i) or the level of inefficiency (u_i). Ignoring such endogeneity can lead to biased and inconsistent parameter estimates. Several approaches have been developed to address endogeneity. These include Corrected Two-Stage Least Squares (C2SLS), which employs standard 2SLS with instruments and corrects the intercept using moments of the 2SLS residuals. Likelihood-based approaches involve specifying a system of equations for the production frontier and the endogenous inputs, and then maximizing a joint likelihood function (Amsler et al., 2016). Another method is the Method of Moments approach which adapts the first-order conditions of the MLE under exogeneity, utilizing instrumental variables for conditions involving endogenous regressors (Amsler et al., 2016). Lastly, the Economic Approach involves estimating the SFM concurrently with the first-order conditions derived from a firm's optimization problem, therefore explicitly modeling input choices. If endogeneity is present, the Jondrow et al. (1982) inefficiency predictor can be modified to $E(u_i|\epsilon_i, \eta_i)$ (where η_i are residuals from the endogenous regressor equations), potentially leading to improved predictions of inefficiency.

3.3 Robust and Conditional Nonparametric Efficiency Analysis

DEA provides a flexible nonparametric benchmark for relative efficiency, but empirical work often goes beyond ranking DMUs and asks whether contextual factors—such as extension services, soil quality, climate, or subsidies—systematically influence performance. This creates a distinct set of statistical challenges that do not arise in the same way in parametric stochastic frontier models. In particular, (i) DEA efficiency scores are estimated objects constructed from a common sample frontier, (ii) they are biased in finite samples, and (iii) they can be highly sensitive to outliers and extreme observations. Robust nonparametric inference therefore typically requires an additional statistical layer on top of the deterministic frontier construction.

Historically, a prevalent approach was a "two-stage" procedure in which DEA scores estimated in the first stage were regressed on environmental variables in a second stage. Simar and Wilson (2007) show that this practice yields invalid inference. Because DEA scores are computed relative to a common estimated frontier, efficiency estimates are serially dependent across DMUs, violating the independence assumptions underlying standard regression methods. Moreover, DEA scores are biased estimators of latent efficiency, implying that naive second-stage regressions also suffer from measurement error in the dependent variable.

The Simar–Wilson framework resolves these issues by specifying a coherent data-generating process for two-stage analysis and implementing a double-bootstrap procedure (Simar and Wilson, 2007). The first bootstrap loop corrects the finite-sample bias in the DEA efficiency estimates. The second bootstrap loop then propagates the dependence structure induced by frontier estimation to obtain valid confidence intervals and hypothesis tests for the parameters in the second-stage regression. Empirical applications in agriculture, such as Latruffe et al. (2008) and Balcombe et al. (2008), illustrate that ignoring these statistical properties can lead to spurious conclusions about the significance and magnitude of efficiency drivers.

A complementary line of work emphasizes robustness in nonparametric frontier estimation. In many agricultural datasets, measurement error, rare weather shocks, or heterogeneous technologies can generate extreme observations that disproportionately shape the DEA/FDH frontier. Robust "partial frontier" estimators mitigate this sensitivity by benchmarking units against a frontier that is intentionally less influenced by extremes (e.g., order-m and order-α frontiers), thereby trading a small amount of approximation for improved stability and interpretability (Daraio and Simar, 2007). This robustification is particularly useful when the objective is comparative performance evaluation rather than exact enveloping of all sample points.

Beyond robustness, Daraio and Simar (2005) and Daraio and Simar (2007) develop a conditional efficiency framework in which environmental variables enter directly into the construction of the attainable set. Rather than treating context only as a second-stage determinant of inefficiency, conditional frontiers evaluate each DMU relative to peers operating under comparable conditions. This approach is well-suited to settings where agro-climatic conditions or institutional environments shift feasible production possibilities and where separability assumptions implicit in conventional two-stage methods may be questionable.

3.4 Efficiency Measurement Under Uncertainty

Traditional efficiency analyses assume a deterministic production environment or treat stochastic elements as statistical noise. Thus, the stochastic frontier method estimates a frontier for production process that is non-stochastic but whose observed outcomes are due to the traditional sources of econometric error.

Most real-world producers, however, operate in environments where stochastc factors that are beyond their control, such as weather and macroeconomic outcomes, affect their realized outcomes. This is peculiarly true for agriculture production whose intrinsic nature is inherently stochastic. Chambers and Quiggin (2000, 2002) develop a generalized framework to understand production decisions and efficiency in such environments. Originating in the work of Arrow (1953), Savage (1954), and Debreu (1959), this approach generalizes the notion of output. Working in the case of a single-output technology, the approach treats production as a random variable, $y = (y_1, y_2, \ldots, y_S) \in \mathbb{R}^S$, where each component y_s corresponds to the specific output that will be realized if a particular state of nature s occurs. Producers make their input decisions (x) ex-ante, before the resolution of uncertainty, selecting a production plan (x, y) from a technology set T that defines feasible combinations of inputs and state-contingent output vectors. The state determining what output level occurs is only determined after (x, y) has been selected by the producer.

Efficiency estimation is complicated by the fact that output is formally an $S-$dimensional vector, but we, as researchers, only observe a one-dimensional scalar outcome that correspond to the state that actually occurs. Econometric methods have been developed to infer or account for these multiple potential realities. One strategy is to empirically characterize distinct production environments for each relevant state of nature. Instead of assuming a single production frontier, this perspective acknowledges that the feasible output set, and thus the benchmark for efficient performance, shifts with the prevailing state (O'Donnell et al., 2010). The analytical task then becomes identifying these differing productive capacities. Efficiency can be evaluated conditional on the specific circumstances a producer encounters. This allows for a clearer distinction between genuine inefficiency and outcomes attributable to unfavorable operating conditions.

Another strategy is reconstructing the ex-ante set of production possibilities. Because only one state of nature is realized and observed for each producer, this method aims to infer the unobserved elements of the state-contingent output vector $(y_1, \ldots, y_S)$. This is achieved by leveraging observable variables (such as detailed weather data, agronomic indicators, or relevant market indices) that are correlated with the underlying, unobservable states of nature. By relating these proxies to observed outputs, it is possible to empirically approximate the range of outcomes the producer might have achieved across various states, creating a more comprehensive basis against which ex-ante efficiency can be assessed (Chavas, 2008;

Nauges et al., 2011; Bokusheva and Baráth, 2024).

A third strategy is to utilize economic valuation principles to understand the state-contingent technology through its dual representation. The premise is that rational producers or competitive markets implicitly assign values (marginal costs or shadow prices) to achieving outputs in different states of nature. Econometric analysis attempts to estimate these state-dependent values. Once the cost structure of producing the vector of state-contingent outputs, $c(w, y)$, is identified, this provides a complete (dual) description of the underlying state-contingent technology. Efficiency is then evaluated by comparing a producer's actual resource use or costs against the minimum cost frontier defined by this comprehensive technological representation (Chambers, 2007; Chambers and Quiggin, 2010; Shankar, 2015).

Data Envelopment Analysis (DEA) has also been adapted for uncertain environments. Event-specific DEA partitions data based on observable events that proxy for states of nature. This allows for the estimation of distinct frontiers for each event or directly incorporates environmental variables into the model structure (Chambers et al., 2011; Skevas and Serra, 2016). Another approach involves the direct elicitation of producers' ex-ante state-contingent output expectations. Instead of inferring them, researchers ask producers to specify their expected outputs $(y_1^i, \ldots, y_S^i)$ for their current input use x^i under various predefined scenarios representing different states of nature. These elicited data are then used to construct DEA frontiers that explicitly reflect the producers' own perceptions of their state-contingent production possibilities, against which their efficiency can be assessed (Chambers et al., 2015a; Serra et al., 2014).

4 Recent Contributions in Agriculture

This section reviews methodological extensions in productivity and efficiency analysis that have been motivated by the distinctive biological, environmental, and institutional features of agricultural production.

4.1 Heterogeneity and Technology Gaps

4.1.1 The Metafrontier Framework

Agricultural production is characterized by substantial heterogeneity across regions, farm types, and producer groups due to differences in agro-climatic conditions, resource endowments, production scale, and institutional environments (Alem et al., 2019). When such heterogeneity is ignored, standard pooled frontier models that impose a common technology across all producers may conflate differences in feasible production opportunities with differences in performance.

To address this issue, the metafrontier framework evaluates performance relative to both group-specific technologies and a common reference technology that envelopes them (T^*) (Battese and Rao, 2002; Battese et al., 2004; O'Donnell et al., 2008). The metafrontier represents the boundary of the union of group-specific technology sets while each group frontier characterizes the best practice attainable within a particular production environment (Figure 3). This approach allows production units operating under different technologies to be compared without imposing a homogeneous production frontier.

Define group-specific production processes $\{T_k\}_{k=1}^{G}$ and a metafrontier T^* that envelopes their union. For each group k, the group output is

$$Y_k(x) \equiv \{y \in \mathbb{R}_+^M : (x, y) \in T_k\},$$

and define the group output distance function

$$D_o^k(x, y) \equiv \inf\{\theta > 0 : \; y \in \theta \, Y_k(x)\}.$$

The metafrontier output set $Y^*(x) \equiv \{y : (x, y) \in T^*\}$ and its output distance function

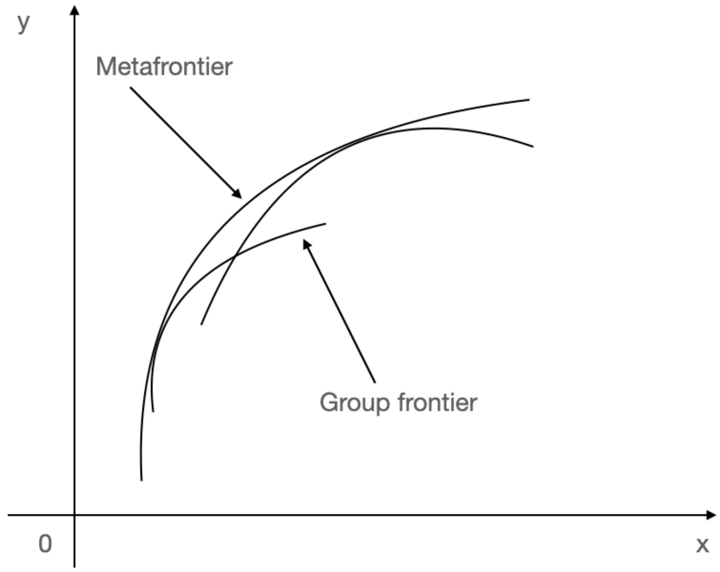

Figure 3: Metafrontier and group frontier.

$D_o^*(x, y)$ are defined in a parallel manner. Output-oriented technical efficiency relative to the group frontier is measured by the group output distance,

$$TE_k(x, y) \equiv D_o^k(x, y) \in (0, 1].$$

The technology gap ratio (the distance between the group frontier and the metafrontier at (x, y)) can be expressed as the ratio of distance functions:

$$TGR_k(x, y) \equiv \frac{D_o^*(x, y)}{D_o^k(x, y)} \in (0, 1].$$

Overall performance relative to the metafrontier is

$$TE^*(x, y) \equiv D_o^*(x, y) = TE_k(x, y) \times TGR_k(x, y).$$

Within this decomposition, TE_k captures how effectively a producer uses the technology available within its environment, whereas TGR_k measures the distance between that environment's frontier and the best-practice technology represented by the metafrontier. In agricultural settings, TE_k is often interpreted as (conditional) managerial performance while TGR_k reflects structural technology gaps associated with production conditions, scale, infrastructure, or institutions. A growing empirical literature applies this framework to identify structural gaps across observable groups. For exmaple, Alem et al. (2019) apply a stochastic metafrontier model to Norwegian dairy farms and identify significant regional gaps.

4.1.2 Endogenous Groupings and Estimation

A practical question is how groups (and therefore T_k) should be defined. Many studies impose groups ex ante (for example, regions or farm types). However, in agricultural data, the relevant regimes often reflect production systems that cut across administrative boundaries and are only partially captured by observable indicators. Latruffe et al. (2023) treat the technology regime as a latent variable. Using a latent-class stochastic frontier combined with a metafrontier, they identify intensive and extensive dairy technologies endogenously. This reveals that performance gaps are often driven by the specific production system adopted rather than the country of location.

Endogenizing regimes, however, often makes estimation more demanding, especially in hierarchical panel metafrontier models that separate farm- and group-level components or distinguish persistent from transitory performance. In such cases, maximum-likelihood approaches may require numerically intensive integration over latent components. Skevas (2025a) proposes a Bayesian estimator that uses simulation-based inference (MCMC with data augmentation) instead of numerical integration. This improves tractability in applications that separate farm-level and group-level components and distinguish persistent from transitory performance differences in panel data.

4.1.3 Alternatives to Discrete Segmentation

Beyond discrete group frontiers, two alternatives are prominent: measurement-based standardization that reduces spurious segmentation, and models that allow technology to vary continuously across space.

Pieralli (2017, 2022) argues that apparent technological differences often arise from treating environmental endowments, such as soil, as simple scalars rather than multidimensional economic inputs. By utilizing separability theory to construct quality-adjusted indices, researchers can retain a single frontier that accounts for non-monotonic environmental characteristics, thereby reducing spurious technology segmentation. This measurement-based logic extends to other resources; Vrachioli and Tzouvelekas (2022) argues that water scarcity should be modeled as a quality-adjusted effectiveness issue, proposing an "effective water use" input that disentangles degradation from volumetric shortages.

Finally, technological heterogeneity need not be discrete. Technology and constraints may vary continuously across space because climate, soils, institutions, and information diffusion change gradually. This motivates models in which the frontier is spatially structured rather than segmented into independent groups. For example, Alemayehu and Kumbhakar (2025) model technology and innovation as spatially correlated processes so that frontier parameters vary with location.

4.2 Dynamics, Adjustment, and Long-Run Productivity

Agricultural production decisions are intertemporal. Biological cycles, quasi-fixed inputs, and irreversible investments tie current decisions to future outcomes and generate adjustment costs. As a result, a static frontier benchmark that evaluates each period in isolation can misinterpret transitional behavior as inefficiency. Dynamic extensions of efficiency and productivity analysis address this benchmark mismatch by incorporating intertemporal constraints and trade-offs, either through productivity decompositions or through explicitly dynamic frontiers.

4.2.1 Index-number decompositions

A first strand employs index-number productivity decompositions to measure productivity change along observed adjustment paths without imposing period-by-period optimality (Chambers and Diakosavvas, 2022). These methods compare observed input–output bundles across time and allow the production frontier to shift between periods, decomposing productivity growth into components associated with technical change, efficiency change, and scale or mix effects. Transitional dynamics, such as gradual capital adjustment, learning-by-doing, and recovery from adverse shocks, are reflected in measured efficiency and scale components while frontier shifts capture technological progress.

Empirically, O'Donnell (2012) shows that while technical change drives long-run U.S. productivity, short-run fluctuations largely reflect adjustment costs. Recent work refines this by modeling weather as a stochastic input: Chambers and Pieralli (2020) and Chambers et al. (2020) find that weather shocks and slowing adaptation, rather than declines in technological capability, drive TFP variability in the U.S. and Australia. Because this refinement treats weather as a stochastic input, it requires consistent weather indexing to cleanly separate shocks from technical change (Chambers et al., 2022). Beyond weather, Sheng (2025) decomposes agricultural productivity growth into efficiency change, technological progress, and capital deepening, allowing technical change to be non–Hicks-neutral. Applying this framework to OECD agriculture, Sheng (2025) shows that frontier shifts are driven mainly by labor-augmenting capital deepening rather than by improvements in technical efficiency.

4.2.2 Dynamic frontiers with quasi-fixed inputs

A second strand introduces dynamic frontier models that incorporate intertemporal constraints into efficiency measurement. These approaches extend SFA and DEA by allowing for quasi-fixed inputs, capital accumulation, or lagged production effects.

Using dynamic profit frontiers, Ang and Oude Lansink (2018) show that short-run inefficiency in Belgian dairy farming emerged endogenously during capital adjustment, while

Ang and Kerstens (2023) document similar transitional productivity losses in French meat-processing firms. These findings highlight that part of what appears as "inefficiency" in the short run can be an endogenous consequence of adjustment. Complementary econometric work makes this distinction explicit by separating persistent from transient components of performance. Multi-component stochastic frontier models such as the Generalized True Random Effects (GTRE) framework partition deviations from the frontier into unobserved producer-specific heterogeneity (α_i), persistent inefficiency ($\bar{u}_i$), transient inefficiency (u_{it}), and idiosyncratic noise (v_{it}). The log-linearized production relationship is specified as:

$$\ln(y_{it}) = \ln[f(x_{it}; \beta)] + \alpha_i + v_{it} - (\bar{u}_i + u_{it}).$$

Kumbhakar et al. (2014) show that failing to account for unobserved producer effects can overstate persistent inefficiency using Norwegian grain farm data. This line of research has been refined to address endogeneity in input choices. For example, Bokusheva et al. (2023) analyze French crop farms and exploit moment conditions derived from the likelihood first-order conditions to obtain consistent parameter estimates in the presence of simultaneity between inputs and unobserved shocks. Alongside Minviel and Sipiläinen (2021), they find that a substantial share of measured agricultural inefficiency is persistent, which suggests that long-run structural constraints and factor rigidities are often more important drivers of performance gaps than temporary behavioral errors.

Parallel evidence from nonparametric methods points in the same direction. Dynamic DEA models relax the period-by-period benchmark by linking consecutive observations through intertemporal constraints so that inefficiency is assessed relative to a feasible adjustment path rather than a sequence of static frontiers. In this setting, part of the "inefficiency" measured by static DEA can reflect omitted lagged production effects. For example, Skevas et al. (2012) show that once contemporaneous and lagged productivity impacts of pesticides are modeled explicitly, estimated inefficiency declines relative to static benchmarks.

Beyond reduced-form adjustments, structural dynamic efficiency frameworks embed production decisions within intertemporal optimization problems and establish a direct link between efficiency measures and optimal dynamic behavior. For instance, Serra et al. (2011) derive a dual relationship between a dynamic directional distance function and the underlying value function using the Hamilton–Jacobi–Bellman equation. Their method allows inefficiency to be assessed relative to a dynamically optimal benchmark rather than a sequence of static frontiers.

4.3 Environment, By-production, and Circularity

Agricultural activities simultaneously produce marketable outputs alongside environmental by-products such as nutrient surpluses, greenhouse gas emissions, and other pollutants. When undesirable outputs are treated as freely disposable or ignored altogether, efficiency and productivity measures become distorted. The problem is the mis-specification of the underlying production technology. Under free disposability, reductions in undesirable outputs are assumed to be costless, which contradicts physical laws and agronomic reality. In agriculture, pollution abatement often requires additional inputs, changes in production practices, or reductions in desirable output. As a result, conventional frontier models may misclassify environmentally responsible behavior as technical inefficiency and overstate true performance gaps.

4.3.1 The By-Production Framework and Material Balance

Early theoretical contributions emphasize the need for production representations that explicitly link pollution generation to the production of desirable outputs. By-production and material-balance approaches provide a framework for modeling this jointness. In these models, production is represented as the intersection of a "good-output" technology and a "bad-output" technology, linked by accounting constraints that reflect conservation of matter and energy (Førsund, 2009; Murty and Russell, 2018; Førsund, 2018). Undesirable outputs are treated as weakly disposable and arise unavoidably from production activities rather than as independent choice variables.

Empirical agricultural applications illustrate how incorporating by-products changes conclusions relative to conventional frontiers. Using a by-production framework that treats nutrient surpluses as undesirable outputs, Hoang and Coelli (2011) show that efficiency scores for agricultural producers decline once pollution abatement is incorporated into the technology. Dakpo and Oude Lansink (2019) apply a material-balance-based DEA model to account for nitrogen emissions in European agriculture and find that conventional efficiency measures overstate performance when environmental constraints are ignored. Extending this logic to greenhouse gas emissions, Ang et al. (2022) demonstrate that a multi-equation by-production framework is required to derive consistent shadow prices for emissions, revealing productivity dynamics in the polluting sub-technology that conventional TFP measures fail to capture.

The same by-product framework naturally extends to settings where by-products are not simply "waste" but can be transformed and re-used. Wang et al. (2023) model crop residue recycling as a joint production decision in which output expansion is inseparable from the generation and re-use of organic by-products. In this setting, recycling activities expand the feasible set by transforming undesirable by-products into productive inputs while mitigation efforts alter the joint generation of outputs and emissions. Accounting for these circular processes changes efficiency rankings relative to conventional technologies that treat residuals as purely waste.

Recent work has also extended the by-production framework to explicitly value environmental services. Bostian and Lundgren (2022) advocate for integrating "mitigating" outputs—such as biodiversity or carbon storage—into productivity measures by leveraging environmental-economic accounting principles. Kunwar et al. (2025) adapt the framework to model carbon sequestration as a desirable output alongside undesirable GHG emissions. Using a DEA approach on field-level data for Illinois corn growers, they derive shadow prices for these carbon services to assess the economic feasibility of sustainable practices. They find that the shadow value of the carbon sequestration service provided by cover crops significantly exceeds current incentive payments, suggesting that voluntary carbon markets may currently undervalue the sequestration benefits of adoption.

Diakosavvas and Chambers (2022) formalize this framework within the TFP context by deriving an environmentally adjusted Solow residual that respects the material balance principle, showing that conventional measures can confound technical change with changes in by-product generation. Complementing this index-number perspective with a frontier-based approach, Ancev et al. (2023) develop distance-function-based productivity indicators defined on a technology that explicitly includes undesirable by-products. Their method treats desirable outputs and undesirable outputs jointly, measuring productivity change relative to an environmentally constrained technology. This avoids attributing changes in environmental performance to "pure" productivity growth and preserves a tight link between productivity measurement and the underlying production technology.

4.3.2 Estimation and Identification Strategies

An econometric difficulty in by-production (joint) technologies is that the same latent factors (for example, management ability, technology choice, and measurement error) often move both market output and pollution in the same direction. If the good-output and bad-output equations are estimated as conditionally independent, this shared unobserved heterogeneity is pushed into the error terms, creating correlated composite errors and potentially confounding how policy and farm characteristics map into "economic" versus "environmental" inefficiency. Skevas (2025b) deals with this problem by estimating a multi-equation by-production stochastic frontier and linking the composite errors from the desirable- and undesirable-output equations with a copula, allowing the dependence structure to be estimated rather than assumed away. In their Dutch dairy application, this matters because variables such as subsidies and stocking density can influence production performance and environmental performance through different channels; accounting for cross-equation dependence helps avoid attributing correlation-driven differences to inefficiency effects, and it can change both coefficient estimates and the resulting farm efficiency rankings (Skevas, 2025b).

While Skevas (2025b) focuses on the statistical dependence between equations, a fundamental structural challenge in by-production models is identifying which constraint—production or abatement—actually binds for a given producer. Standard single-frontier models conflate these distinct processes, potentially biasing shadow price estimates. Addressing this, Yan and Chambers (2025) develop a Full-Information Maximum Likelihood (FIML) framework with endogenous regime switching to jointly estimate separable production and abatement frontiers. Applied to Chinese hog farms, their model endogenously sorts producers into "production-bound" or "abatement-bound" regimes based on their input mix and external constraints. They find that single-frontier models systematically misestimate abatement costs by averaging across these regimes, whereas the regime-switching approach recovers the true marginal land cost of pollution control for farms where the abatement constraint is binding.

Even when the joint structure is correctly specified, empirical performance can still hinge on how flexibly the frontier is approximated in high-dimensional environmental settings. A complementary methodological direction is to relax the restrictive frontier estimator while preserving the by-production structure. Guillén et al. (2025) retain the by-production representation of joint production but replace the conventional DEA estimator with efficiency analysis trees, which allows the frontier to be approximated by flexible, nonlinear partitions of the input–output space. By embedding machine learning estimation within the by-production framework, their approach relaxes functional form restrictions while preserving the axiomatic properties of the technology.

4.4 Spatial and Institutional Constraints

4.4.1 Spatial Dependence and Spillovers

Agricultural production is spatially dependent. Farms share aquifers, pest populations, and information networks. This violates the independence assumption inherent in standard frontier models. Methodological extensions address this by embedding a spatial weight matrix, $\mathbb{W}$, into the production structure to characterize the connectivity between producers i and j.

In the parametric context, spatial spillovers are modeled by specifying the technology or inefficiency as a function of spatially lagged variables. A general spatial stochastic frontier can be represented as:

$$y_i = f(x_i; \beta) + \rho \sum_{j \neq i} \omega_{ij} y_j + v_i - u_i$$

where ρ captures the spatial autoregressive spillover of output (or productivity). Hailu and Deaton (2016) adapt this logic to a stochastic input distance function, modeling inefficiency u_i as dependent on local farm density.

Skevas (2023) embeds spillovers directly in productivity measurement by constructing TFP growth components that include a spatial interaction term defined over a neighbor network (via a spatial weights matrix). Conceptually, each farm's observed productivity change is decomposed into (i) "internal" sources—technical change and efficiency change driven by its own input–output adjustments—and (ii) an "external" component that captures exposure to neighbors' technology and performance through the spatially weighted environment, so that part of measured TFP growth is attributed to diffusion effects rather than the farm's own innovation. In contrast, Skevas and Oude Lansink (2020) do not impose a spatial structure inside the DEA estimator itself: they first compute dynamic efficiency scores from a dynamic DEA technology and then test whether these scores exhibit spatial dependence by applying Moran's I, showing that dynamic inefficiency is systematically clustered across neighboring farms rather than being idiosyncratic noise.

4.4.2 Institutional Constraints and Policy Analysis

Institutional constraints, ranging from subsidies to organizational structures, also act as external restrictions on the feasible input–output set, distorting the shadow prices faced by producers. The standard methodological approach is to condition the inefficiency distribution on policy variables. If u_i represents technical inefficiency, it is specified as $u_i \sim D^+(\mu_i, \sigma_u^2)$, where the location parameter $\mu_i = q_i'\delta$ is a function of policy vectors q_i. Latruffe et al. (2017) employ this conditional approach to show that subsidies do not have a monotonic effect on efficiency; rather, their impact is mediated by investment constraints and risk preferences.

Bernini and Galli (2024) extend the conditional-inefficiency logic by allowing policy exposure to be spatially coupled. They estimate a spatial stochastic frontier for Italian agriculture and model both direct subsidy effects and spillovers from spatially lagged subsidies. Their results indicate that subsidy exposure is associated with stronger environmental performance while the economic effect is more mixed. Spatial spillovers are statistically important: within-province spillovers reduce inefficiency whereas inter-province spillovers increase economic inefficiency but decrease environmental inefficiency.

Beyond public policy, institutional constraints also arise from private organizational arrangements that change incentives and effort so that observed performance gaps can reflect agency costs rather than technology. In developing economies, the choice between vertical integration (plantations) and contract farming (outgrowers) alters the incentive structure. Wendimu et al. (2017) address this utilizing a generalized nonparametric kernel regression that admits categorical variables:

$$E(y_i|x_i, q_i^c, q_i^d) = m(x_i) + r(q_i^c, q_i^d)$$

where q represents discrete institutional indicators (factory vs. outgrower). By avoiding restrictive functional forms, they isolate the impact of moral hazard inherent in wage-labor contracts, demonstrating that "inefficiency" in factory farms is structurally driven by agency costs rather than technological inferiority.

4.5 Uncertainty Management

Standard frontier models evaluate performance relative to an ex post benchmark that conditions on realized outcomes. This assumes that producers can condition their decisions on the state of nature that occurs. In agriculture, however, production choices are made ex ante, before uncertainty regarding weather, pests, or prices is resolved. Input use therefore reflects trade-offs across multiple possible states rather than optimization with respect to the single realized outcome. From this perspective, a producer may optimally choose a plan that appears inefficient ex post because it hedges against unfavorable states or manages uncertainties.

Several empirical strategies address the mismatch between ex ante decision-making and ex post efficiency benchmarks. One approach evaluates efficiency conditional on realized events rather than imposing a single pooled frontier. Chambers et al. (2011) develop event-specific DEA models in which production frontiers differ across observable states of nature, such as weather conditions or pest pressure. A complementary strategy relies on dual, cost-based representations of state-contingent technologies. Chambers (2007) and Chambers and Quiggin (2010) estimate cost structures under state-contingent production and recover shadow prices associated with outcomes across states of nature, showing that input choices that appear excessive under deterministic benchmarks may be consistent with cost minimization once uncertainty is accounted for. Another line of work reconstructs the ex ante state-contingent technology itself. Chambers et al. (2015b) and Serra et al. (2014), combine production data with elicited expectations or structural restrictions to recover vectors of potential outputs associated with different states of nature. Efficiency is then assessed relative to the feasible ex ante choice set rather than the single realized outcome.

5 Conclusion

This paper reviews the main methodological foundations of efficiency and productivity analysis and synthesizes recent advances motivated by core features of agricultural production. We highlight how standard parametric and nonparametric frontier benchmarks derive their meaning from the maintained representation of technology and the behavioral and physical constraints embedded in it. Much of the recent agricultural literature can be interpreted as refining the benchmark to better match empirical realities: accounting for technology heterogeneity, incorporating intertemporal adjustment and biological lags, imposing physically coherent treatments of undesirable outputs and joint production, and modeling spatial dependence and institutional constraints that shape feasible production sets.

Several implications follow. (i) Frontier choice is a substantive modeling decision: different representations of disposability, dynamics, and joint production define different benchmarks and therefore different notions of inefficiency. (ii) Decompositions that separate within-technology performance from technology gaps, and transient deviations from persistent components, are useful for credible interpretation in agricultural settings. (iii) Empirical credibility requires valid inference with the dependence structures induced by frontier methods and by spatial, temporal, and institutional linkages. (iv) The methodological frontier is moving toward combining flexibility with structure (for example, integrating machine-learning estimators with axiomatic technology restrictions) so that nonlinearities can be captured without abandoning the economic meaning of the production set.

Several directions appear important for future research. First, integrative frameworks that jointly accommodate dynamics, uncertainty, environmental jointness, and spatial interactions would improve comparability across studies and strengthen economic interpretation. Second, greater attention to identification and valid inference is essential for moving from descriptive measurement to credible explanation, including endogeneity, selection, and finite-sample bias in frontier and second-stage settings. Third, combining flexible function approximation (including machine-learning tools) with economically motivated shape restrictions offers a promising route to capturing nonlinearities without sacrificing the structural meaning of the production set. Overall, the field is shifting from reporting a single efficiency score toward delivering empirically credible decompositions that explain why performance differs across producers, space, and time under the constraints that are central to agriculture.

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