Abatement, Weather, and Land: An Input-Requirement Model of Shadow Prices and Trade-offs in Livestock Production

Abstract

Intensive livestock production is a major source of ammonia nitrogen emissions, yet the environmental effectiveness and land implications of alternative manure management systems remain unclear. We develop a land-requirement model to evaluate pollution abatement, land use, and internal mitigation costs in commercial hog farms regulated under China's national pollution control program. The stochastic frontier specification treats land as the only freely disposable input and allows congestion among traditional inputs, abatement activities, meat output, and NH_3-N. Using detailed farm-level data on production, abatement infrastructure, emissions, and weather, we estimate input-requirement frontiers for soil/fishpond integration, aerobic anaerobic treatment, and biogas systems. From these frontiers we derive marginal rates of substitution between meat and NH_3-N and among all inputs, along with nominal shadow prices for all inputs and outputs. These findings show that technology mandates alone cannot ensure environmental performance. Policies that target verified reductions in NH_3-N per unit of output and account for land and climate constraints are more likely to deliver cost-effective pollution control.

Keywords: pollution management, stochastic frontier analysis, abatement shadow prices, hog farming, input requirement model

JEL Codes: D24, Q12, Q52, Q53, Q57

1 Introduction

Livestock farming is fundamental to sustainable food systems. It supplies essential protein, supports rural development, and anchors major agricultural value chains. Yet its polluting by-products impose significant environmental pressures. Intensive livestock operations contribute to nitrogen losses, eutrophication, and atmospheric NH_3 emissions (Gerber et al., 2013; Njuki and Bravo-Ureta, 2015). Understanding these joint relationships is central to environmental management when land constraints and nutrient balances shape pollution outcomes.

China's hog sector offers an important example. China is the world's largest hog producer. Regulatory reforms under its Twelfth Five-Year Plan (2011-2015) mandated upgrades to manure storage, treatment, and recycling systems on commercial hog farms. Implementation of these reforms generated a detailed national data set on input use, abatement structures, manure handling, and pollution outcomes in hog production. Complementary agronomic and environmental studies show that manure storage, recycling, and land-application practices affect nutrient losses and environmental risks in Chinese livestock systems (Bai et al., 2016, 2019; Hu et al., 2023). This combination of regulatory pressure and rich micro-level data makes the Chinese hog sector a natural setting for evaluating how environmental technologies shape pollution reductions and resource use.

Although China's regulatory context is distinctive, the underlying mechanisms it confronts – manure surpluses, land scarcity, and heterogeneous abatement strategies – mirror challenges faced by livestock producers globally. We study the Chinese case to gain insights relevant to worldwide environmental management of intensive livestock systems.

Our analysis begins by specifying a model of livestock production that reflects these biophysical realities. Livestock systems jointly produce meat, manure, and pollutant emissions. Abatement activities, such as storage and treatment, use resources that might otherwise support production. In our model, we treat land as the only freely disposable input or output. This helps capture its ecological role as the spatial sink that assimilates manure nutrients. Standard production models often assume that *all inputs and outputs are freely disposable.* Producers can expand inputs or reduce outputs without affecting the feasibility of production plans. That assumption overlooks important technological constraints that confront livestock production where emissions are a necessary by-product of production. Undesirable outputs, such as contaminated water, require costly remediation. And agricultural inputs can cause congestion and emissions when applied in excessive amounts.

Shephard (1970) early recognized that undesirable outputs are not freely disposable. Subsequent theoretical work (Førsund 1998; Pethig 2006; Murty et al. 2012; Murty and Russell 2022) showed that production models involving pollution must incorporate the joint nature of desirable and undesirable outputs. These theoretical results reflect fundamental constraints in agriculture and livestock systems, where emissions are physically linked to production processes. Material-balance concerns further emphasize the need for production models to recognize the physical entanglement of emissions, abatement activities, and productive inputs in production systems (Ayres and Kneese, 1969; Pethig, 2006; Coelli et al., 2007; Murty et al., 2012; Murty and Russell, 2022). Empirical studies show that imposing free disposability of undesirable outputs can bias efficiency measurement and understate production costs (Hailu and Veeman, 2001; Dakpo et al., 2016). These issues are especially salient in livestock systems where manure nutrients and NH_3 emissions are inseparable from animal growth and feed conversion.

We use this conceptual framework to develop an empirical model that captures the linkages between production, abatement, and emissions. We estimate a stochastic land-requirement frontier that incorporates desirable output, conventional inputs, abatement activities, and NH_3-N emissions. This approach provides a flexible framework for evaluating how different abatement strategies influence land demands and pollution outcomes.

Our empirical approach is econometric, but it is related conceptually to nonparametric environmental-efficiency methods. Data envelopment analysis (DEA) has been widely used to evaluate pollution-generating production systems (Chung et al., 1997; Färe et al., 2001). Applications to livestock production show how nutrient management and manure handling affect measures of environmental performance (Huang and Wang, 2017). Studies of pig finishing demonstrate how benchmarking can identify cost-saving nitrogen mitigation options (Van Meensel et al., 2010). These methods do not separate random shocks from inefficiency and thus may struggle in settings characterized by weather variation, disease risk, and stochastic feed quality.

Stochastic frontier analysis, on the other hand, addresses these concerns by jointly modeling technical inefficiency and econometric error (Aigner et al., 1977; Meeusen and van Den Broeck, 1977; Kumbhakar and Lovell, 2003). Stochastic frontier models have incorporated undesirable outputs into production or distance functions (Reinhard et al., 1999; Coelli et al., 2005, 2007). Research has documented substantial heterogeneity in environmental performance across farms and different production environments (Fernandez et al., 2005; Mamardashvili et al., 2016). Recent work highlights the importance of nutrient management, abatement practices, and climatic conditions in shaping environmental efficiency (Adenuga et al., 2019; Skevas et al., 2018; Mozahid et al., 2025).

Our estimated econometric model provides quantifiable information on the physical trade offs between hog production, pollution generation, abatement efforts, and other inputs in the production process. Thus, our analysis extends an existing literature that infers implicit (shadow) costs of pollution and abatement while quantifying environmental trade-offs (Coggins and Swinton, 1996; Färe and Grosskopf, 1998). Previous work shows that treating pollution as freely disposable distorts shadow prices and misrepresents marginal abatement costs (Färe et al., 2005; Dakpo et al., 2016). Livestock applications have estimated shadow prices for nitrogen and phosphorus surpluses (Adenuga et al., 2019; Mamardashvili et al., 2016). Research on greenhouse gas mitigation derives marginal abatement cost curves for dairy systems (Lengers et al., 2014; Huber et al., 2023) and evaluates the financial implications of emission-reduction strategies (Cantillon et al., 2024). By estimating marginal rates of substitution among land, inputs, abatement activities, and NH_3-N emissions within a unified structure, our model links these insights to a regulatory context where manure management is central to compliance and environmental performance.

Our analysis has direct implications for environmental management in livestock systems. By quantifying the pollution responses, we show how producers internalize the trade-offs between production and environmental control under regulatory constraints. The empirical analysis suggests that abatement investments mandated by policy yield limited environmental gains. Hence, compliance-oriented approaches should be complemented by strategies that integrate land availability and its nutrient-assimilative capacity. These findings speak to global concerns about designing effective manure-management regulations in regions where land is scarce and environmental pressures are mounting.

The remainder of the paper proceeds as follows. Section 2 presents our proposed input-requirement framework. Section 3 introduces the data set constructed from a policy intervention in China targeting non-point source pollution. Section 4 describes the empirical strategy and presents the regression results. Section 5 further discusses the empirical results' implications. Section 6 concludes.

2 Technology

Let

$$T = \Big\{(l, x, a, y, b) : (l, x, a) \text{ can produce } (y, b)\Big\}, \tag{1}$$

be a closed and nonempty production technology set. Here, l represents the farm's land, x denotes a vector of traditional inputs, a is a vector of abatement activities, y represents the desirable (good) output, and b represents undesirable (bad) output.

We separate l from the other traditional inputs because we are confident in asserting that land is freely disposable for our data set. That is, an increase in l does not inhibit the farm's productive capacity. We cannot make the same claim for other traditional inputs in Chinese agricultural production processes. It has been widely documented that Chinese farmers overuse inputs such as fertilizer and pesticide (Wu et al., 2018; Ren et al., 2021). And, since Shephard (1970)'s classic work, it is well-known that free disposability of output is problematic in the presence of undesirable outputs. Coelli et al. (2007) show that material-balance matters further exacerbate such concerns.

Therefore, we remain agnostic about the disposability properties of (x, a, y, b) in T and treat them as empirical matters to be resolved by the data. Aside from non-emptiness and closedness, our only other restriction on T is that $(l,x,a,y,b) \in T \Rightarrow (\hat{l},x,a,y,b) \in T$ for $\hat{l} \geq l$ (*free disposability of land*). That assumption ensures that we can develop a land-requirement function, $L(x,a,y,b)$ that is a *function representation of* T. That is,

$$(l,x,a,y,b) \in T \Leftrightarrow L(x,a,y,b) \leq l$$

where

$$L(x,a,y,b) \equiv \inf\left\{l : (l,x,a,y,b) \in T\right\}. \tag{2}$$

That $(l,x,a,y,b) \in T \Rightarrow l \geq L(x,a,y,b)$ follows directly by the definition of $L(\cdot)$. To establish that $(l,x,a,y,b) \in T \Leftarrow l \geq L(x,a,y,b)$, note that by definition $(L(x,a,y,b),x,a,y,b) \in T$ and then invoke free disposability of land. Thus, $L(\cdot)$ characterizes T in the sense that knowing the former is equivalent to knowing the latter, and the reverse.

That our empirical representation models a *function representation* of T is crucial to our goal of obtaining empirical estimates of shadow prices for the various production activities. For smooth technologies, local shadow prices are derived from the empirical gradient of the function representation of T. They are intimately linked to the basic economic concepts of rates of substitution, marginal productivities, and rates of transformation.

The level set

$$I(y,b) \equiv \{(x,a,l) : l = L(x,a,y,b)\}$$

corresponds to the usual notion of a production isoquant for fixed (y,b). Measures of input substitutability and complementarity are determined in the smooth case by the signs of the slopes of $L(x,a,y,b)$ in (x,a). The respective *marginal rates of substitution* (MRS) between land and (x,a) are given in the smooth case by, respectively, $\frac{\partial L(\cdot)}{\partial x}$ and $\frac{\partial L(\cdot)}{\partial a}$. The MRS between x and a, is determined by $-\frac{\partial L(\cdot)}{\partial a}/\frac{\partial L(\cdot)}{\partial x}$ (the MRS between elements within x or a are defined analogously). If the MRS is negative implying, for example, that increasing x requires reducing l to maintain production holding (a,y,b) constant, l and x are *substitutes.* If the MRS is positive, the inputs are *complements.*

We use the term "marginal rate of substitution" guardedly. Our model differs from more traditional approaches that treat all "inputs" as freely disposable by only requiring land to be freely disposable. The more traditional approach implies that all inputs have positive marginal products. That, in turn, requires all inputs to be substitutes. Thus, the traditional identification of an isoquant's slope with a marginal rate of *substitution* is apt. Our approach allows for "input congestion" and potentially negative marginal products. "Inputs" can be complementary to one another within the set $I(y, b)$. We retain the phrasing, however, because of its familiarity.

The processes of transforming inputs into outputs and outputs into one another are captured by

$$P = \{(x, a, y, b, l) : l = L(x, a, y, b)\}$$

For example, the *marginal product* of land in producing y is $(\frac{\partial L(\cdot)}{\partial y})^{-1}$ and the marginal product of input x in producing y by $-\frac{\partial L(\cdot)}{\partial x} / \frac{\partial L(\cdot)}{\partial y}$. The *marginal rate of transformation* of b into y is $-\frac{\partial L(\cdot)}{\partial b} / \frac{\partial L(\cdot)}{\partial y}$. We again caution that the terms *marginal products* and *rates of transformation* are used for mnemonic purposes.

The various marginal rates give *real shadow prices* to producers. As real prices, they are expressed in quantity units rather than currency units. To facilitate the conversion to currency units, we define the producer's *hog revenue function* as the maximum revenue that can be generated from hog production given (l, x, a, b) as

$$R(p, l, x, a, b) \equiv \underset{y}{sup}\{py : l \leq L(x, a, y, b)\},$$

where $p > 0$ is the price of y. $R(p, l, x, a, b)$, as is well-known, is positively homogeneous and convex in p.[1] It also satisfies McFadden's Lemma (Chambers, 1988). In the smooth case:

$$\frac{\partial R(p, l, x, a, b)}{\partial p} = y(p, l, x, a, b),$$

where $y(p, l, x, a, b)$ is the revenue maximizing supply. In our single good-output case, one might conjecture that the revenue maximizing supply represents an asymmetric production function (Chambers, 1988). That is only appropriate, however, if T exhibits global

[1] In our empirical application, y is a scalar so that $R(p, l, x, a, b)$ is linear in p.

free disposability of the good output, which we have not imposed.

Using standard programming arguments and the envelope theorem, we identify the (internal) *nominal shadow prices* of (l, x, a, b) with the slopes of $R(p, l, x, a, b)$ in those variates. For an interior solution $(y > 0)$ to the revenue maximization problem, these shadow prices can be recaptured from $L(x, a, y, b)$ in the smooth case as $\frac{\partial R(\cdot)}{\partial l} = p(\frac{\partial L(\cdot)}{\partial y})^{-1}$; $\frac{\partial R(\cdot)}{\partial x} = -p(\frac{\partial L(\cdot)}{\partial y})^{-1}\frac{\partial L(\cdot)}{\partial x}$; $\frac{\partial R(\cdot)}{\partial a} = -p(\frac{\partial L(\cdot)}{\partial y})^{-1}\frac{\partial L(\cdot)}{\partial a}$; and $\frac{\partial R(\cdot)}{\partial b} = -p(\frac{\partial L(\cdot)}{\partial y})^{-1}\frac{\partial L(\cdot)}{\partial b}$. Accordingly, the MRS can be equivalently expressed as the negative ratios of the corresponding shadow prices.

3 Data

The study relies on detailed records from a national effort to reduce non-point source pollution from commercial Chinese hog farms. The regulation was implemented from 2012 to 2014 within China's Twelfth Five Year Plan and required any farm with more than five hundred pigs to install or improve waste treatment systems. These upgrades generated detailed records on input use, herd size, abatement structures, and pollution emissions. Our main pollution indicator is NH_3-N. It is the only nitrogen based measure that is consistently recorded across the survey and therefore serves as our environmental outcome.

Farms report their addresses, which allows us to link them to local climate conditions. Weather data come from NOAA station records. We interpolate summer and winter temperature and precipitation for each farm using inverse distance weighting in ArcGIS to reflect spatial gradients in rainfall and heat. These variables capture climatic pressures that may influence manure handling, nutrient losses, and the performance of treatment systems.

Each farm is assigned to one of three groups based on its primary abatement approach. The first group combines land application and fishpond integration. The second group uses aerobic and anaerobic treatment structures. The third group relies on biogas based digestion units. Figure 1 presents descriptive patterns for these groups using spatial maps and box plots.

Panel (a) displays the distribution of meat output production. Farms in the aerobic and anaerobic group dominate the central and eastern provinces and often exceed one thousand tons of meat per year. Several clusters reach two thousand tons or more. Biogas farms are the most common, they also concentrated in the east but appear farther north. All groups display a wide production range, with many farms producing several thousand tons. Land and pond farms show smaller scales on average. They are spread more evenly across the south and southeast. This spatial variation suggests that treatment choice is closely related to production scale and market access.

Panel (b) plots the distribution of NH_3-N emissions. The pattern strongly mirrors output scale and shows the tight link between production intensity and environmental load in the sector. It is common for these farms to release values between ten and thirty tons. A small set of extremely large farms exceeds fifty tons.

Panel (c) provides a broader look at land use, abatement inputs, traditional inputs, and weather conditions across farms. Land and pond farms report the largest land areas, in many cases more than one thousand square meters. This reflects their use of land and water bodies as part of the treatment process. Aerobic and anaerobic farms show large capacities for engineered structures. Many farms operate lagoons, digesters, clarifiers, and oxidation ponds with volumes that reach several thousand cubic meters. Biogas farms have sizable digester tanks and slurry pits. They also maintain substantial storage volume for biogas residues. These contrasts illustrate the distinct engineering requirements of each treatment strategy.

Traditional inputs such as labor and herd sizes also vary. Aerobic and anaerobic farms employ more workers, invest more capitals, and maintain larger herds. Biogas farms have comparable herd sizes but often operate with fewer workers, reflecting a more mechanized system. Land and pond farms keep smaller herds and use fewer labor inputs, which aligns with their more extensive production practices.

Climate conditions show meaningful differences. Group 1 farms face higher rainfall in both summer and winter. This pattern indicates greater exposure to runoff risks and may explain why these farms rely on broader land areas. Group 3 farms are located in warmer regions, with many sites reporting high summer temperatures. Warmer climates support efficient anaerobic digestion and may help stabilize biogas production.

Together, these descriptive patterns show a sector where abatement technologies, production scale, spatial conditions, and environmental outcomes are tightly interlinked. The diversity across groups emphasizes the importance of modeling production and abatement jointly. It also highlights the role of climate and geography in shaping the environmental performance of livestock systems.

4 Empirical Strategy and Estimation Results

Our empirical approach centers on the land requirement function $L(x, a, y, b)$, which links production scale, abatement activities, pollution loads, and site specific climatic conditions to the minimum land area needed to operate a hog farm. Land is a binding biophysical constraint in livestock production. Regulatory compliance, nutrient loading limits, and treatment infrastructure all require space. The land requirement framework therefore provides a direct environmental interpretation of farm behavior that is well suited for evaluating the spatial consequences of alternative abatement systems.

To estimate this relationship, we adopt a log-log stochastic frontier specification using maximum likelihood. The functional form allows each coefficient to be interpreted as an elasticity, while the frontier structure separates the minimum land required for production and abatement from additional land use attributable to management inefficiency. This is especially relevant in livestock systems where engineered facilities, containment structures, and ecological treatment designs impose physical space requirements. The stochastic frontier approach captures these minimum spatial demands more accurately than linear or average response models.

The empirical model is:

$$
\begin{aligned}
\ln l_i &= \ln L_i(x,a,y,b) \\
&= \alpha_0 + \sum_{n=1}^{N} \beta_n \ln x_{ni} + \sum_{m=1}^{M} \gamma_m \ln a_{mi} + \delta_y \ln y_i + \delta_b \ln b_i + \delta_w w_i + v_i - u_i
\end{aligned}
\tag{3}
$$

where l denotes land utilization, y and b denote the meat output (good output) and NH_3-N emission (bad output), respectively, x_j, represents the traditional inputs, a_m denotes the m-th abatement activity, w represents the weather factors for a specific farm, including the winter and summer average precipitation and temperature for the year of abatement installation. The composite error term consists of a symmetric noise component, $v_i \sim i.i.d.\ \mathcal{N}(0,\sigma_v^2)$, and a one sided non negative term, $u_i \sim i.i.d.\ \mathcal{N}^+(0,\sigma_u^2)$, which reflects departures from the minimum land frontier implied by the production and abatement technologies.

Because we specify the dependent variable (l) and the independent variables in logarithmic terms, the parameters of the log-log specification have the natural interpretation of the percentage change in land use required to balance a 1 percent change in the use of the independent variable. Thus, they are "elasticities". They are not to be confused, however, with Allen, Uzawa, Morishima, or Shadow Elasticities of substitution that measure the curvature of the input isoquants. Moreover, such measures, which are framed in terms of convex and freely disposable technology models, have limited applicability in more general settings.

Table 1 reports the land-requirement function's coefficient estimates for the three farm groups. Meat output exhibits a positive elasticity across all groups, confirming that expanding production raises the spatial needs of the operation. The elasticity is largest for aerobic and anaerobic systems, where a one percent increase in output requires more than a one percent increase in land. This may reflect stricter pollution controls or weaker substitution across inputs. These systems depend on multi stage treatment structures and strict containment, which increases the spatial requirements per unit of output. Land and pond systems exhibit a smaller but still meaningful elasticity, while biogas farms display the least land sensitivity to output changes, consistent with the compact, engineered nature of anaerobic digestion units.

Pollution outcomes also influence land use. The coefficients on NH_3-N emissions are all negative, indicating that lowering emissions requires additional land for treatment, storage, or retention. Cleaner operation thus imposes spatial costs, especially for engineered systems where containment volumes and safety margins scale with treatment intensity. The near zero coefficient in Group 1 reflects the diffuse, landscape based nature of soil and fishpond systems.

Traditional inputs follow intuitive patterns. Labor raises land requirements in the land and pond farm group, reflecting the labor intensive nature of manure handling and ecological treatment. Capital investment shows little association with land in any farms, likely because mandated upgrades do not alter the physical layout of farms. Herd size has a strong negative elasticity in the aerobic and anaerobic group, suggesting scale related spatial efficiencies in housing and waste management.

Abatement inputs have diverse and system-specific effects on land use. A consistent finding is that manure storage increases require increasing land use. It introduces spatial burdens in all groups. In contrast, liquid storage is land saving. That likely reflects liquid storage's role in consolidating waste. Group 1 benefits from integrated fishponds. They can reduce land needs by recycling waste into aquaculture. Digester and drainage systems also save land in this group, showing the spatial efficiency of integrated ecological designs. In Group 2, aerobic lagoons reduce land use. Anaerobic lagoons, however, increase land demand. That may suggest higher spatial costs for containment and microbial processing. Group 3 shows a consistent land penalty from all digester-related systems. These include digestion tanks, storage facilities, and filtration units, all of which are land-intensive. Biogas systems may offer environmental gains but carry clear spatial trade-offs.

Climate variables further shape spatial requirements in livestock production. Abatement success depends on climate compatibility. Because weather falls beyond the producer's control, abatement is inherently stochastic. Summer warmth and rainfall reduce land requirements. That seems to reflect enhanced nutrient cycling, faster decomposition, or more efficient manure management under warm, wet conditions. Cold or wet winters, in contrast, raise land use. These effects may reflect slower decomposition, limited evaporation, and restricted land application in colder months. The effects are strongest in Groups 2 and 3, as engineered systems still depend on external climate conditions.

The results as a whole show that land plays a central role in determining the environmental performance of livestock farms. Cleaner production and more advanced treatment systems often require additional land, and the magnitude of this requirement varies across ecological, engineered, and energy recovery systems. Understanding the input-requirement model and the land implications of abatement strategies is therefore essential for designing effective policy instruments in livestock environmental management.

5 Analysis

This section examines the marginal rates of substitution (MRS) and shadow prices implied by the land-requirement estimates in Table 1. We focus on two types of derived measures. First, the full MRS matrices, reported in Appendix Table A1, contain all marginal relationships among the twenty-one inputs and two outputs for the three farm groups. From these matrices, we extract the output MRS between NH_3-N and meat output and map its spatial distribution in Panel (a) of Figure 2. This MRS quantifies the additional pollution generated per incremental ton of meat and provides a direct measure of production-pollution trade-offs across regions and abatement systems. Second, the shadow prices of inputs and emissions, summarized in Table 2, measure the implicit economic value of constrained resources. We highlight the spatial distribution of NH_3-N shadow prices in Panel (b) of Figure 2 to show how the marginal cost of emissions varies across farms.

The discussion is organized into four parts. Subsection 5.1 presents a detailed interpretation of the output MRS and the spatial patterns in Panel (a) of Figure 2. Subsection 5.2 revisits the broader marginal relationships in Appendix Table A1 and is divided into two parts: one on the effects of abatement and weather variables on output performance, and one on substitution and complementarity patterns among inputs. Subsection 5.3 examines the shadow prices of inputs and emissions using Table 2 and the spatial patterns in Panel (b) of Figure 2. Subsection 5.4 concludes by drawing general implications for abatement system design and environmental policy.

5.1 Output Marginal Rates of Substitution

We first examine the marginal trade-off between meat production and NH_3-N emissions. Holding land fixed, the marginal rate of substitution (transformation) between NH_3-N and meat output for farm i is defined as the additional pollution generated when meat output increases by one unit while land use remains at its current level. Totally differentiating the land-requirement function yields $\left.\frac{\partial b_i}{\partial y_i}\right|_{l_i} = -\frac{\partial L_i/\partial y_i}{\partial L_i/\partial b_i}$. Under the log-log specification in (3), the partial derivatives are $\partial L_i/\partial y_i = (\delta_y L_i)/y_i$ and $\partial L_i/\partial b_i = (\delta_b L_i)/b_i$, so that $\mathrm{MRS}_{b,y,i} = \left.\frac{\partial b_i}{\partial y_i}\right|_{l_i} = -\frac{\delta_y}{\delta_b}\frac{b_i}{y_i}$. Although the coefficient ratio $-\delta_y/\delta_b$ is common to all farms within a group, the output ratio b_i/y_i varies by location. As a result, spatial heterogeneity in the MRS reflects both the technology embodied in the land-requirement coefficients and the local balance between emissions and meat output. In our maps, this measure is expressed in kilograms of NH_3-N per additional ton of meat, which aligns with standard environmental reporting practices and provides a more policy-relevant scale for comparing marginal pollution burdens across regions and technologies.

Panel (a) of Figure 2 displays the county-level average MRS between NH_3-N and meat output for the three abatement groups. The scale of the trade-off differs sharply across systems. The group using land and fishpond integration exhibits by far the highest values, with an average MRS of about 1,562 kilograms of NH_3-N per additional ton of meat. Farms relying on aerobic-anaerobic treatment show a much lower average of roughly 83 kilograms NH_3-N/ton of meat, while biogas-based farms have the smallest average MRS at approximately 11 kilograms NH_3-N/ton of meat. These differences span more than two orders of magnitude and indicate that, conditional on land, Group 1 generates vastly more pollution per marginal unit of meat than the engineered systems.

These scale differences are consistent with the technological and environmental characteristics documented in the data and empirical sections. Land and pond systems rely on open surfaces and ecological dispersion through soils and water bodies. Such designs provide limited control over volatilization and runoff and are concentrated in wetter climates where rainfall can mobilize nitrogen. In contrast, aerobic-anaerobic systems use multi-stage lagoons and oxidation ponds that capture and treat a larger share of waste streams, while biogas systems employ sealed digesters and slurry management that substantially confine emissions. The much lower output MRS in Groups 2 and 3 therefore reflects both improved process control and, in the case of biogas, the recovery of part of the nitrogen in energy-rich by-products.

Within each abatement group, the spatial distribution of the MRS shows a clear inland-coastal gradient that persists even after controlling for abatement inputs, traditional inputs, herd size, and climatic conditions. Lower MRS values are concentrated along the eastern seaboard, while higher values appear more frequently in central and inland regions. These spatial patterns reflect systematic differences in emissions intensity per unit of meat that remain after conditioning on the covariates in the land-requirement model. This suggests that land-based nutrient dispersion interacts with local production scales and waste-loading pressures in ways that create more severe marginal pollution burdens in inland regions. These may arise from regional differences in operational practices, treatment sequencing, or accumulated waste loads that influence the pollution released per incremental ton of meat.

Overall, the output-to-output MRS maps show that technology choice and location jointly shape the incremental environmental cost of livestock production. Land and pond systems impose very high additional emissions per ton of meat, engineered lagoon systems perform markedly better, and biogas systems offer the lowest marginal pollution burdens, especially in warmer coastal regions. These patterns emphasize the importance of both technological upgrading and spatially differentiated policy when targeting reductions in NH_3-N from commercial hog production.

5.2 Broader Marginal Relationships Among Inputs & Outputs

Beyond the output substitution patterns examined above, the full marginal structure of the production-abatement system can be characterized using the complete MRS matrices reported in Appendix Table A1. These matrices summarize how each input or abatement activity affects meat output and NH_3-N emissions at the margin, and reveal how inputs substitute for or complement one another when holding land use constant. We report its implications for production performance and abatement design into two sets of findings.

Effects of abatement and weather inputs on output performance

The MRS relationships linking abatement activities to meat output and NH_3-N emissions show clear and consistent patterns across groups. Farm area increases meat output and decreases NH_3-N across all groups. This "double-dividend" effect shows land's dual role in promoting desirable production while providing ecological buffering capacity. Liquid storage has a similar dual effect. Its function may stem from better containment and waste separation. Manure storage, however, raises emissions and lacks output gains across all groups. It behaves more as a pollutant reservoir than as a mitigation strategy.

Group-specific abatement strategies reflect divergent environmental and productive roles. Group 1's fishponds and drainage improve both output and environmental performance. These suggest functional ecological integration. Group 2's aerobic lagoons reduce pollution and raise output, but anaerobic ones do not. These fail to curb emissions and seem to bring no production benefit. Group 3's biogas systems also show trade-offs. Slurry and storage components increase emissions and offer little output gain.

Traditional inputs, labor, capital, and herd size, show weaker but consistent patterns. Labor and capital tend to reduce meat output and, in some cases, increase emissions. We conjecture that these congestive effects may arise from regulators' compliance restrictions. However, we also note that other analyses find congestion in Chinese agricultural production.

Weather variables also influence output performance at the margin. Weather variables suggest that summer temperature complements meat output in all groups while substituting for NH_3-N emissions. At the observed margin, additional warmth seems to improve animal metabolism and reduce on-site nitrogen loads through accelerated volatilization. Winter cold does the opposite. It impedes productivity and slows nitrogen dissipation. Rainfall effects differ by season. Summer rain supports meat output, possibly through pasture regeneration or thermal relief. Winter rain worsens outcomes, especially for Group 1, likely due to runoffs. In sum, climate conditions shape both emissions and productivity, and thus adaptive and seasonal policies are essential.

Substitution and complementarity among inputs

The MRS matrix in Appendix Table A1 also provides a detailed view of how inputs interact in maintaining production. These values show how one input offsets another to maintain output in the case of substitutes and how they vary together for complements.

Farm area and manure storage are complements for all groups. This pairing reinforces the earlier point: manure storage uses valuable resources. It raises emissions and does not enhance output, yet is used alongside land. In contrast, liquid storage substitutes for land. It offers flexibility and reduces the need for spatial buffering. Farms also pair liquid and manure storage, suggesting redundant strategies.

Group-specific abatement strategies exhibit variation. Group 1 suggests effective input trade-offs. Digesters, fishponds, and drainage substitute for land and for each other. This reflects system flexibility and integration within an ecological infrastructure. Group 2 shows limited trade-offs. Only aerobic lagoons substitute for land. Other components (anaerobic lagoons, hydrochloric acid ponds, and oxidation ponds) are stacked with limited marginal returns. This design seems to limit flexibility and raise costs. Group 3's slurry, biogas storage, clarifiers, and filtration are all complementary to land or liquid storage. This land-using relationship seems to bring no observable environmental dividends or efficiency. This reduces flexibility and raises coordination costs without guaranteed emission reductions. This is potentially due to compliance mandates or policy-driven technology adoption without performance targeting.

Traditional inputs also fail to offset land or liquid storage. Labor and capital scale with abatement inputs rather than substitute for them. This reflects a pattern of overusage and input congestion. Instead of boosting efficiency, they seem to increase reliance on space and containment.

Summer heat and rain tend to substitute for land and liquid storage. Favorable growing weather conditions can reduce spatial needs for productivity and waste dispersion. Winter weather does the reverse. It raises demand for space and containment. This finding suggests that climate-sensitive planning should become part of abatement design.

Overall, the marginal relationships documented in Appendix Table A1 reveal how the structure of abatement systems shapes both their operational flexibility and their environmental footprint. Ecological designs allow a wider set of input adjustments, engineered lagoon systems permit only selective substitution, and biogas facilities operate through tightly linked components that raise spatial dependence. These contrasts show that the environmental consequences of livestock production arise not only from the scale of activity but also from the interaction between technological structure and local operating conditions. Understanding these interactions is essential for interpreting variation in pollution outcomes and for anticipating how different systems respond to constraints on land, storage capacity, or seasonal operating windows.

5.3 Shadow Prices of Inputs and NH_3-N

Shadow prices quantify the marginal values to the producer of varying inputs and outputs. They are derived from the revenue function $R(p, l, x, a, b)$ and expressed in dollars by scaling the marginal rates of substitution by the market price of meat. Formally, the shadow price of any variable z equals $-\partial R/\partial z$, which under the log-log specification is proportional to the elasticity of the land-requirement function with respect to that variable multiplied by the farm's observed scale. These values capture how a marginal increase in an input, an abatement activity, or NH_3-N emissions affects the maximum attainable revenue through its impact on the land constraint. Table 2 reports the group-level averages of these nominal shadow prices.

The shadow price of NH_3-N differs markedly across abatement systems. Group 1 exhibits a value close to zero, implying that a marginal increase in emissions yields almost no revenue gain through the land-requirement channel. Combined with the very high output-to-emissions MRS documented in Subsection 5.1, this suggests a weak technological linkage between emissions and productive capacity in land-pond systems. Emissions do not meaningfully relax the land constraint, nor do they enhance output at the margin. Group 2 has a moderately higher pollution shadow price, and Group 3 displays the highest. In engineered aerobic-anaerobic and biogas systems, a marginal increase in NH_3-N releases more room within the land constraint when evaluated in revenue terms, reflecting tighter connections between waste flows, containment structures, and spatial requirements.

The spatial distribution in Panel (b) of Figure 2 complements these group-level differences. Lower pollution shadow prices are concentrated along the eastern coast, while inland southeastern counties exhibit substantially higher values, with levels declining again in the far south and southwest. Because the model conditions on inputs, abatement structures, and weather, these gradients reflect systematic differences in how production systems translate marginal emissions into revenue-relevant relief of the land constraint. High shadow prices indicate that local systems obtain more revenue per unit of marginal emissions, while low values indicate that emissions are of little marginal value in production.

The shadow prices of land and abatement inputs further illuminate the technological structure of each group. Land carries a large positive shadow price in all groups, highest in Group 3, consistent with the spatial intensity of biogas systems. Manure storage has negative shadow prices across all groups, indicating that additional manure storage capacity reduces revenue by raising emissions or forcing reliance on spatially burdensome structures. Liquid storage receives positive valuations everywhere, consistent with its ability to consolidate waste streams and reduce pressure on the land requirement.

Group-specific components behave differently. In Group 1, fishponds and drainage systems carry positive shadow prices, reflecting their contribution to reducing land pressure while supporting production. Group 2 assigns positive value only to aerobic lagoons;

anaerobic lagoons and several chemical or oxidation components have negative valuations, indicating poor marginal performance relative to their spatial cost. Group 3 exhibits negative shadow prices for digester slurry, biogas storage, and filtration units, suggesting that additional investment in these tightly linked components increases spatial or operational burdens without improving revenue at the margin.

Traditional inputs display consistent patterns. Labor and capital generally carry negative shadow prices, particularly in Groups 2 and 3, implying mild congestion when these systems expand personnel or fixed investment relative to their land constraints. Pig stocks are weakly positive in the engineered systems but negative in Group 1, consistent with the limited absorptive capacity of land-pond systems.

Overall, the shadow price results reveal marked differences in how abatement systems convert marginal variations in inputs and emissions into revenue through their interaction with the land constraint. These patterns show the role of technological structure in determining the economic and environmental performance of livestock operations.

5.4 Lessons Learnt

The results point to several considerations for the design and evaluation of livestock abatement strategies. A first observation is that the environmental and spatial implications of abatement depend strongly on the structure of the underlying system. Technologies that are integrated into the broader production environment, such as ecological dispersion in Group 1 or engineered containment in Groups 2 and 3, generate markedly different marginal relationships. The very low pollution shadow price for Group 1 combined with its high output-to-emissions MRS indicates that land-pond systems make weak use of emissions at the margin and create high incremental pollution loads. By contrast, the tighter coupling among waste flows, containment units, and land requirements in Groups 2 and 3 yields higher pollution shadow prices and lower marginal emissions per unit of output. The variation in these patterns shows that abatement outcomes depend less on technological complexity than on how the system aligns with the spatial and operational conditions of production.

A second lesson concerns the allocation and targeting of abatement investments. Our results suggest caution in the blanket imposition of abatement infrastructure without performance verification. Our evidence suggests that many advanced systems were installed for regulatory compliance rather than real environmental performance. Groups 2 and 3 include several examples where expensive components do not improve output or reduce pollution. This leads to poor substitutability, higher pollution shadow prices, and more congested input use. The system meets regulatory requirements but not economic or environmental goals. Policies that reward installation may encourage overbuilding and crowd out more appropriate strategies.

Third, the role of weather and seasonal variation shows that abatement strategies must be adaptive. Summer and winter conditions generate distinct patterns of input substitutability and output performance. A static or rigid infrastructure is unlikely to perform optimally year-round. Weather uncertainty should be incorporated into policy design. Current policies rarely address this issue or incentivize seasonal flexibility. This oversight can create inefficiencies in management practices. Policies or technical guidelines that encourage adaptability, or that recognize the role of seasonal operating windows, may offer more resilient environmental outcomes.

Taken together, these findings suggest that effective environmental management requires policies that consider how abatement systems interact with production scale, spatial constraints, and operating conditions. Shadow prices and substitution patterns reveal where technologies function as intended and where they impose additional burdens. These insights suggest a key principle for abatement policy: incentivize environmental effectiveness and not installation *per se*. Programs should prioritize outcome-based evaluation criteria, such as verified reductions in NH_3-N emissions per unit of output, over input-based compliance metrics. Funding and technical support should focus on flexibility and adaptability. Abatement strategies should also match the farm's layout, climate, and resource availability.

A broader lesson emerges from the analysis. Sustainable livestock systems require alignment, between technology, production, and environmental needs. Mandates alone cannot ensure efficiency or fairness. Shadow prices and substitution patterns reveal where abatement works, where it fails, and why. A one-size-fits-all policy may waste resources and worsen inefficiencies in the livestock sector.

6 Concluding Remarks

This paper evaluates the environmental performance of commercial hog farms by combining a flexible land-requirement model with detailed information on abatement activities, production scale, pollution outcomes, and climatic conditions. The framework departs from conventional production models by relaxing global free disposability, treating land as the only freely disposable input, and allowing congestion among traditional and abatement inputs. By modelling abatement activities as distinct and economically costly inputs, rather than as unobserved residuals, we advance the empirical treatment of pollution mitigation in agricultural-environment systems. These features enable explicit empirical identification of shadow prices, substitution relationships, and the marginal costs of reducing NH_3-N emissions within heterogeneous treatment systems.

Our methodological innovation lies in the application of the land requirement function while relaxing disposability of other inputs, outputs, and by products. The model estimates how much land is needed to support a given bundle of inputs, outputs, and pollution. Our model offers a realistic characterization of the trade-offs involved in environmentally sustainable livestock production. This is especially important in hog farming in China or other developing countries where input overuse and environmental constraints are widespread.

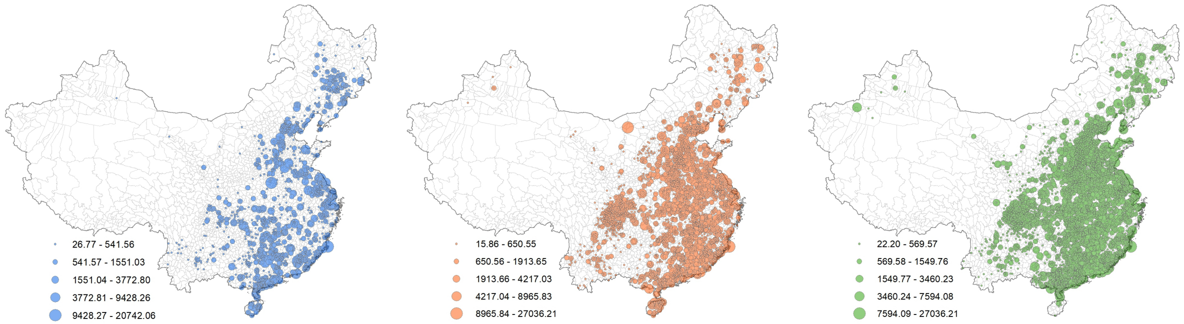
Our empirical application demonstrates how abatement infrastructures interact with production requirements and spatial constraints across three dominant systems in China: land-pond integration, aerobic-anaerobic treatment, and biogas digestion. The stochastic frontier estimates reveal that these systems differ substantially in their land requirements, pollution-output trade-offs, and marginal valuations of inputs. Some abatement components function as genuine land-saving or pollution-reducing technologies. Others impose additional spatial burdens or contribute little to environmental improvement. Seasonal weather patterns further modify these relationships and highlight the importance of climate compatibility in abatement system design.

The results suggest an important implication for environmental management: technology choice matters, but so does alignment between abatement design, production scale, spatial conditions, and local climate. Regulatory programs that reward installation rather than verified pollution reduction risk encouraging overbuilt or poorly matched systems. Shadow prices and marginal substitution patterns show that several widely subsidized technologies deliver weak environmental returns at the margin. In contrast, systems that integrate abatement within ecological or engineered processes yield clearer environmental gains and more efficient land use.

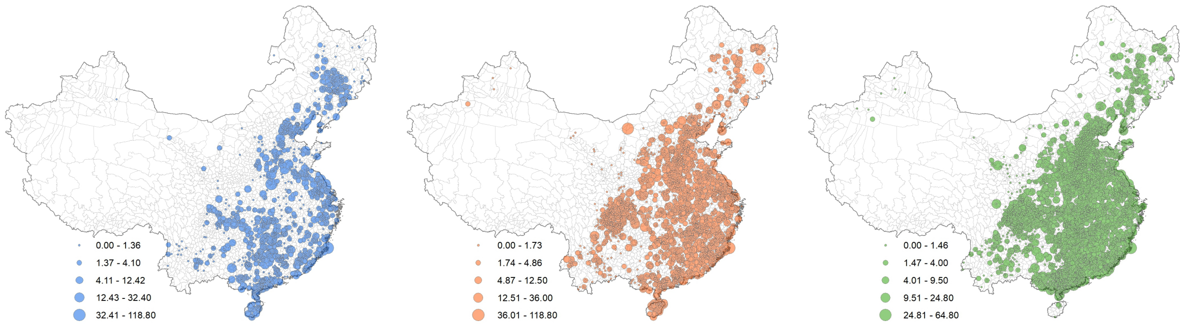
By demonstrating how land requirements link production, pollution, and abatement, this study provides a quantitative basis for more effective policy targeting. The approach identifies where environmental improvements are most feasible, where space or climate impose binding constraints, and where investments yield limited benefits. These insights can inform the design of performance-based subsidies, climate-aware guidelines, and integrated environmental management strategies that better support pollution control in livestock agriculture.

Overall, the land-requirement framework offers a practical and policy-relevant tool for evaluating pollution mitigation in livestock systems. It shows the need for abatement strategies that are technologically functional, spatially compatible, and environmentally effective, and it contributes evidence for aligning agricultural development with sustainable environmental management.

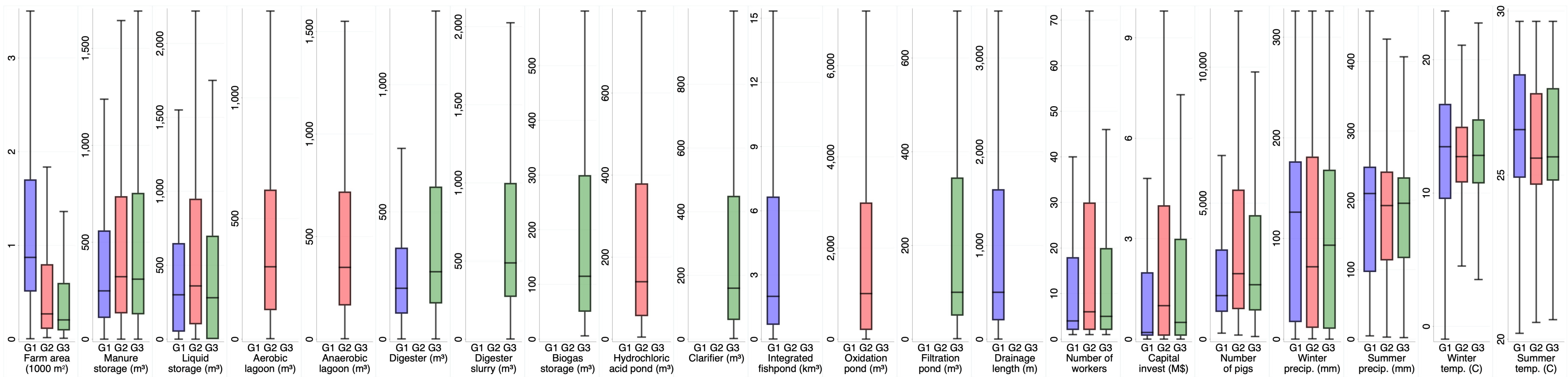
Figure 1: Summary statistics for farms using land-pond integration, aerobic-anaerobic treatment, and biogas-generation strategies



(a) Spatial distribution of meat output (tons) for the three abatement groups (mainland China)



(b) Spatial distribution of NH_3-N (tons) for the three abatement groups (mainland China)



(c) Box plots for land, abatement inputs, traditional inputs, and weather variables

Table 1: Land requirement function with abatement inputs

l.Farm Area	**Group 1**	**Group 2**	**Group 3**
Efficiency Score	0.998	0.999	0.999
y.Meat output (ton)	0.468***	1.133***	0.342***
	(0.068)	(0.054)	(0.035)
b.NH3-N (ton)	-0.00129	-0.0567**	-0.144***
	(0.033)	(0.021)	(0.020)
a1.Manure storage (m^3)	0.0211***	0.0132**	0.00406
	(0.005)	(0.005)	(0.003)
a2.Liquid storage (m^3)	-0.00177	-0.00716*	-0.00843***
	(0.004)	(0.003)	(0.002)
a3.Aerobic lagoon (m^3)		-0.00995**	
		(0.003)	
a4.Anaerobic lagoon (m^3)		0.0390***	
		(0.003)	
a5.Digester (m^3)	-0.000374		0.0189***
	(0.003)		(0.004)
a6.Digester slurry (m^3)			0.0212***
			(0.002)
a7.Biogas storage (m^3)			0.0637***
			(0.007)
a8.Hydrochloric acid pond (m^3)		0.00232	
		(0.007)	
a9.Clarifier (m^3)			0.000720
			(0.005)
a10.Integrated fishpond (km^2)	-0.0550***		
	(0.011)		
a11.Oxidation pond (m^3)		0.00181	
		(0.003)	
a12.Filtration pond (m^3)			0.0278**
			(0.009)
a13.Drainage system length (m)	-0.00600		
	(0.006)		
x1.Number of workers	0.0402***	0.00331	0.000790
	(0.009)	(0.007)	(0.006)
x2.Capital investment (Million $)	0.00272	0.00383	0.00516
	(0.008)	(0.008)	(0.005)
x3.Thousands of pigs	0.0478	-0.806***	-0.0253
	(0.077)	(0.059)	(0.042)
Summer Precipitation (mm)	-0.0000277	-0.00275***	-0.00179***
	(0.000)	(0.000)	(0.000)
Summer Temperature (C)	-0.000312	-0.0213***	-0.00960***
	(0.003)	(0.003)	(0.003)
Winter Precipitation (mm)	-0.000355*	0.00233***	0.00164***
	(0.000)	(0.000)	(0.000)
Winter Temperature (C)	0.000680	0.0217***	0.00593*
	(0.003)	(0.003)	(0.002)
Constant	-2.179***	0.741**	-1.074***
	(0.353)	(0.250)	(0.274)
N	**3772**	**6214**	**12273**

The inefficiency effect is modeled as following a half-normal distribution, and the parameters are estimated using maximum likelihood estimation. Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: The three groups adopt the following methods, respectively: (1) soil/fishpond integration, (2) aerobic-anaerobic waste treatment, and (3) biogas production.

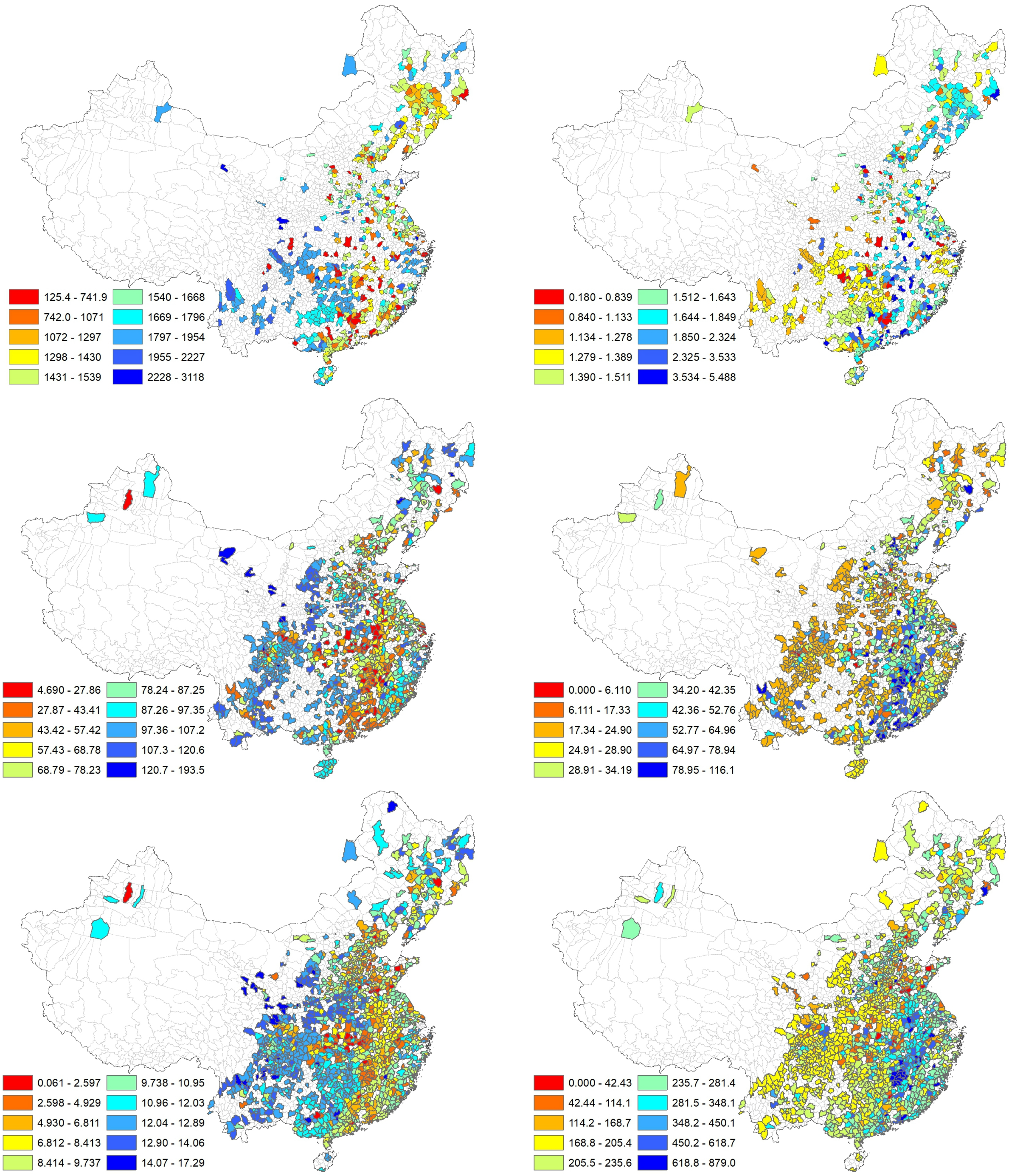
Table 2: Shadow Prices by Group from Land Requirement Models

Shadow Price ($)	**Group 1**	**Group 2**	**Group 3**
y.Meat output (ton, Market Price)	2482.699	2484.674	2496.746
	(0.793)	(1.133)	(0.647)
b.NH3-N (kg)	1.699	35.976	256.565
	(0.011)	(0.256)	(1.036)
l.Farm Area (1000 m^2)	670.058	660.119	2105.142
	(14.094)	(10.221)	(23.464)
a1.Manure storage (m^3)	-4.384	-1.476	-1.277
	(0.477)	(0.157)	(0.095)
a2.Liquid storage (m^3)	0.906	2.886	10.767
	(0.056)	(0.135)	(0.320)
a3.Aerobic lagoon (m^3)	–	6.705	–
	–	(0.206)	–
a4.Anaerobic lagoon (m^3)	–	-13.829	–
	–	(0.529)	–
a5.Digester (m^3)	0.300	–	-1.553
	(0.017)	–	(0.148)
a6.Digester slurry (m^3)	–	–	-53.940
	–	–	(1.014)
a7.Biogas storage (m^3)	–	–	-224.513
	–	–	(3.345)
a8.Hydrochloric acid pond (m^3)	–	-2.946	–
	–	(0.063)	–
a9.Clarifier (m^3)	–	–	-2.472
	–	–	(0.036)
a10.Integrated fishpond (km^2)	94.583	–	–
	(2.534)	–	–
a11.Oxidation pond (m^3)	–	-2.109	–
	–	(0.049)	–
a12.Filtration pond (m^3)	–	–	-98.626
	–	–	(1.450)
a13.Drainage system length (m)	12.143	–	–
	(0.347)	–	–
x1.Number of workers	-64.487	-3.002	-2.146
	(1.210)	(0.045)	(0.022)
x2.Capital investment (Million $)	-2.801	-2.014	-8.342
	(0.059)	(0.043)	(0.108)
x3.Number of pigs	-0.032	0.215	0.022
	(0.000)	(0.001)	(0.000)

Note: Standard errors in parentheses below the estimated means are calculated as $SE = s/\sqrt{N}$, where s is the sample standard deviation of the marginal relation across farms and N is the number of observations. Each marginal relation can be tested against the null hypothesis that it equals zero.

Shadow prices are expressed in U.S. dollars and converted to 2012 real values using the GDP deflator.

Figure 2: Marginal Rates of Substitution and Shadow Prices for NH_3-N (mainland China)



(a) MRS: NH_3-N per additional unit of meat (b) Shadow prices for NH_3-N ($)

Notes: **Panel (a)** shows the marginal rate of substitution (MRS) between NH_3-N (kg) and meat output (ton) for farms adopting soil and fishpond, aerobic-anaerobic, and biogas abatement strategies in mainland China during the 2012-2014 Livestock Regulation. This MRS is defined as pollution generated per additional unit of meat. The complete MRS matrix across all inputs and outputs appears in Table A1 in the appendix. **Panel (b)** shows NH_3-N shadow prices. Shadow prices for other inputs and outputs are reported in Table 2.

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Online Supplementary Appendix

Appendix Table A1 reports the full marginal rate of substitution (MRS) matrices for the three abatement groups. Each matrix contains all pairwise marginal relationships among the twenty-one inputs and the two outputs, derived from the land-requirement estimates in Table 1. These values summarize how a marginal change in any variable must be offset by changes in other variables to keep land use fixed. The matrices provide the structural foundation for the marginal relationships discussed in Section 5.

For any pair of variables z_j and z_k, the MRS holding land constant is $\text{MRS}_{j,k,i} = \left.\frac{\partial z_j/\partial z_k}{\partial L_i/\partial z_k}\right|_{l_i=0} = -\frac{\partial L_i/\partial z_k}{\partial L_i/\partial z_j}$, where the derivatives are taken with respect to the land-requirement function $L(x, a, y, b)$. Under the log-log specification in (3), these derivatives become $\frac{\partial L_i}{\partial z_j} = \frac{\theta_j L_i}{z_{j,i}}$, where θ_j is the elasticity associated with z_j (for example, β_n for traditional inputs, γ_m for abatement inputs, and δ_y and δ_b for outputs). Substituting these expressions yields

$$\text{MRS}_{j,k,i} = -\frac{\theta_k}{\theta_j}\frac{z_{j,i}}{z_{k,i}},$$

which shows that spatial variation in the MRS arises from both the elasticity ratios and the observed levels of the variables. This framework applies to all entries in the matrices, including the output-to-output MRS examined in Subsection 5.1 and the input-to-output and input-to-input interactions interpreted in Subsection 5.2.

The results in Appendix Table A1 should be viewed as the detailed arithmetic underlying the broader patterns reported in the main text. The large dimensionality of the matrices makes them unsuitable for inclusion in the body of the paper, but they offer a comprehensive reference for readers interested in the full marginal structure of the production-abatement system. The spatial MRS maps in Figure 2 and the substitution patterns summarized in Section 5 are derived directly from these entries.

Table A1: Marginal Relation between Inputs and Outputs

Group 1: N=3772	**Outputs (Top/Left)**		**Inputs (Top/Left)**								
	y	b	l	$a1$	$a2$	$a5$	$a10$	$a13$	$x1$	$x2$	$x3$
y.meat output (ton)		1562.399	7.784	-38867.961	568239.510	1305160.400	73.960	16911.002	-402.201	-3893.968	-79628.147
		(4.867)	(1.164)	(1112.051)	(23715.221)	(68955.114)	(1.938)	(2557.402)	(73.443)	(274.286)	(136.048)
b.NH3-N (kg)	0.001		-0.006	26.686	-390.827	-847.411	-0.050	-12.281	0.426	3.103	54.087
	(0.000)		(0.001)	(0.725)	(17.790)	(54.960)	(0.002)	(1.950)	(0.118)	(0.356)	(0.311)
l.Farm Area (1000 m^2)	0.269	-412.181		9186.200	-133371.290	-286019.480	-16.854	-3475.022	135.649	968.020	21476.682
	(0.006)	(11.982)		(184.545)	(4359.909)	(9836.193)	(0.735)	(541.652)	(29.509)	(100.983)	(520.451)
$a1$.Manure storage (m^3)	-0.002	2.449	0.005		23.299	1464.354	0.051	5.370	-0.511	-3.216	-145.783
	(0.001)	(0.270)	(0.001)		(0.662)	(244.227)	(0.003)	(2.930)	(0.155)	(0.304)	(15.086)
$a2$.Liquid storage (m^3)	0.001	-0.470	-0.001	5.572		-415.461	-0.019	-4.347	0.074	0.620	29.905
	(0.000)	(0.031)	(0.000)	(0.676)		(51.468)	(0.006)	(1.371)	(0.017)	(0.045)	(1.784)
$a5$.Digester (m^3)	0.000	-0.177	-0.001	3.924	-62.164		-0.007	-1.167	0.029	0.343	9.431
	(0.000)	(0.013)	(0.000)	(0.232)	(9.867)		(0.002)	(0.585)	(0.007)	(0.107)	(0.578)
$a10$.Integrated fishpond (km^2)	0.038	-54.331	-0.264	1348.412	-17847.892	-48373.621		-475.435	28.630	146.985	3033.771
	(0.001)	(1.535)	(0.044)	(54.615)	(1679.146)	(7644.847)		(103.602)	(6.226)	(21.788)	(81.491)
$a13$.Drainage system length (m)	0.005	-6.986	-0.030	161.124	-2563.912	-5854.146	-0.283		3.201	17.224	386.132
	(0.001)	(0.229)	(0.005)	(6.055)	(240.393)	(845.924)	(0.039)		(0.680)	(2.388)	(11.537)
$x1$.Number of workers	-0.026	38.358	0.142	-896.229	12156.943	25626.042	1.295	286.877		-43.574	-2044.510
	(0.001)	(0.711)	(0.016)	(29.161)	(412.629)	(1101.221)	(0.047)	(68.242)		(1.569)	(38.046)
$x2$.Capital investment (Million $)	-0.001	1.655	0.007	-40.536	549.431	1090.897	0.062	10.883	-0.071		-88.720
	(0.000)	(0.034)	(0.001)	(1.397)	(19.535)	(47.871)	(0.004)	(1.976)	(0.004)		(1.865)
$x3$.Number of pigs	-0.000	0.019	0.000	-0.487	7.307	16.229	0.001	0.198	-0.005	-0.049	
	(0.000)	(0.000)	(0.000)	(0.013)	(0.317)	(0.882)	(0.000)	(0.030)	(0.001)	(0.004)	
Summer Precipitation (mm)	0.024	-0.034	-0.000	0.783	-12.117	-28.043	-0.002	-0.259	0.015	0.082	1.875
	(0.001)	(0.001)	(0.000)	(0.029)	(1.109)	(3.903)	(0.000)	(0.052)	(0.003)	(0.011)	(0.054)
Summer Temperature (C)	0.266	-0.378	-0.002	8.825	-136.580	-316.111	-0.015	-2.917	0.170	0.922	21.128
	(0.007)	(0.012)	(0.001)	(0.324)	(12.497)	(43.992)	(0.002)	(0.591)	(0.036)	(0.124)	(0.616)
Winter Precipitation (mm)	0.303	-0.430	-0.002	10.040	-155.388	-359.641	-0.017	-3.319	0.193	1.048	24.038
	(0.009)	(0.013)	(0.001)	(0.368)	(14.218)	(50.049)	(0.003)	(0.672)	(0.040)	(0.142)	(0.701)
Winter Temperature (C)	-0.581	0.824	0.004	-19.246	297.882	689.442	0.033	6.362	-0.369	-2.010	-46.082
	(0.016)	(0.026)	(0.001)	(0.706)	(27.256)	(95.945)	(0.005)	(1.290)	(0.077)	(0.271)	(1.343)

Note: Standard errors in parentheses below the estimated means are calculated as $SE = s/\sqrt{N}$, where s is the sample standard deviation of the marginal relation across farms and N is the number of observations. Each marginal relation can be tested against the null hypothesis that it equals zero.

Table A1: Continued

Group 2: N=6214	Outputs (Top/Left)		Inputs (Top/Left)									
	y	*b*	*l*	*a1*	*a2*	*a3*	*a4*	*a8*	*a11*	*x1*	*x2*	*x3*
y.meat output (ton)		82.642	8.875	-147439.590	391068.100	106332.380	-31767.661	-17694.218	-911865.690	-29954.502	-7517.557	11766.016
		(0.333)	(0.133)	(2706.126)	(9009.015)	(5313.067)	(881.570)	(2725.843)	(91560.004)	(6919.698)	(570.349)	(19.643)
b.NH3-N (kg)	0.015		-0.127	1974.514	-4810.398	-1399.925	424.416	287.366	12577.699	847.519	112.208	-165.450
	(0.000)		(0.003)	(46.525)	(124.681)	(68.282)	(12.036)	(51.065)	(1284.547)	(253.752)	(10.317)	(1.048)
l.Farm Area (1000 m^2)	0.264	-18.955		30883.718	-79406.654	-21500.967	6128.910	5216.808	158182.600	8316.455	1739.859	-3105.582
	(0.004)	(0.282)		(812.733)	(3227.468)	(2090.106)	(144.491)	(1500.156)	(15761.181)	(1296.238)	(179.114)	(46.337)
a1.Manure storage (m^3)	-0.001	0.036	0.002		4.278	17.274	-9.466	-40.775	-279.281	-13.372	-3.849	6.598
	(0.000)	(0.005)	(0.000)		(0.099)	(4.657)	(1.645)	(28.623)	(87.963)	(6.196)	(1.994)	(0.674)
a2.Liquid storage (m^3)	0.001	-0.072	-0.005	58.312		-46.560	21.777	36.018	702.200	28.006	5.577	-12.934
	(0.000)	(0.004)	(0.001)	(2.999)		(8.962)	(1.649)	(15.992)	(130.831)	(6.769)	(1.268)	(0.592)
a3.Aerobic lagoon (m^3)	0.003	-0.213	-0.013	347.464	-815.294		75.755	64.393	1054.793	22.477	10.190	-32.196
	(0.000)	(0.007)	(0.001)	(12.291)	(45.191)		(2.554)	(24.501)	(167.422)	(5.716)	(1.306)	(1.020)
a4.Anaerobic lagoon (m^3)	-0.005	0.364	0.021	-489.245	1481.497	860.904		-127.978	-4871.550	-147.099	-33.133	61.803
	(0.000)	(0.013)	(0.001)	(23.701)	(187.207)	(132.483)		(27.671)	(668.472)	(33.816)	(4.801)	(2.275)
a8.Hydrochloric acid pond (m^3)	-0.001	0.081	0.005	-136.145	319.912	74.966	-26.382		-565.539	-30.509	-6.317	13.655
	(0.000)	(0.002)	(0.000)	(4.226)	(15.949)	(8.382)	(0.775)		(61.263)	(4.298)	(0.633)	(0.295)
a11.Oxidation pond (m^3)	-0.001	0.058	0.004	-98.490	234.751	48.022	-19.210	-5.910		-24.570	-4.707	9.769
	(0.000)	(0.002)	(0.000)	(3.292)	(12.435)	(6.205)	(0.590)	(0.911)		(3.398)	(0.499)	(0.228)
x1.Number of workers	-0.001	0.088	0.007	-143.742	332.525	75.128	-28.009	-18.983	-658.312		-3.216	13.893
	(0.000)	(0.002)	(0.000)	(3.446)	(8.084)	(4.705)	(0.646)	(7.368)	(72.561)		(0.123)	(0.201)
x2.Capital investment (Million $)	-0.001	0.059	0.005	-99.011	226.417	50.440	-18.470	-8.708	-356.875	-3.172		9.316
	(0.000)	(0.002)	(0.000)	(3.243)	(6.766)	(2.259)	(0.635)	(1.298)	(30.285)	(0.673)		(0.193)
x3.Number of pigs	0.000	-0.007	-0.001	12.593	-32.985	-9.373	2.642	1.522	80.254	2.722	0.644	
	(0.000)	(0.000)	(0.000)	(0.230)	(0.737)	(0.474)	(0.070)	(0.225)	(8.208)	(0.647)	(0.048)	
Summer Precipitation (mm)	1.472	-0.101	-0.007	166.256	-387.396	-92.061	32.490	25.259	739.036	39.014	7.822	-17.043
	(0.030)	(0.002)	(0.000)	(5.038)	(18.983)	(9.982)	(0.937)	(6.878)	(81.784)	(5.204)	(0.760)	(0.359)
Summer Temperature (C)	11.377	-0.778	-0.053	1285.283	-2994.865	-711.704	251.173	195.273	5713.312	301.610	60.468	-131.757
	(0.237)	(0.017)	(0.002)	(38.943)	(146.754)	(77.171)	(7.244)	(53.174)	(632.254)	(40.230)	(5.873)	(2.776)
Winter Precipitation (mm)	-1.248	0.086	0.006	-141.016	328.584	78.085	-27.558	-21.425	-626.840	-33.092	-6.635	14.456
	(0.026)	(0.002)	(0.000)	(4.272)	(16.101)	(8.467)	(0.794)	(5.834)	(69.368)	(4.414)	(0.644)	(0.304)
Winter Temperature (C)	-11.608	0.794	0.054	-1311.366	3055.644	726.148	-256.270	-199.236	-5829.258	-307.731	-61.696	134.430
	(0.241)	(0.018)	(0.002)	(39.733)	(149.732)	(78.737)	(7.391)	(54.253)	(645.085)	(41.047)	(5.992)	(2.832)

Note: Standard errors in parentheses below the estimated means are calculated as $SE = s/\sqrt{N}$, where s is the sample standard deviation of the marginal relation across farms and N is the number of observations. Each marginal relation can be tested against the null hypothesis that it equals zero.

Table A1: Continued

Group 3: N=12273	**Outputs (Top/Left)**		**Inputs (Top/Left)**										
	y	*b*	*l*	*a1*	*a2*	*a5*	*a6*	*a7*	*a9*	*a12*	*x1*	*x2*	*x3*
y.meat output (ton)		10.707	3.126	-175848.380	82054.642	-28354.194	-13083.926	-88.648	-27820.406	-166.449	-16234.435	-1493.155	115979.580
		(0.022)	(0.138)	(2089.015)	(1416.130)	(458.270)	(304.812)	(7.191)	(3180.843)	(28.089)	(2678.011)	(70.373)	(137.867)
b.NH3-N (kg)	0.103		-0.323	17207.684	-7946.440	2821.927	1118.942	9.579	2420.375	19.413	3023.220	165.691	-11720.433
	(0.001)		(0.015)	(214.939)	(149.839)	(51.775)	(29.390)	(0.760)	(195.840)	(2.728)	(778.400)	(10.616)	(39.873)
l.Farm Area (1000 m^2)	0.840	-8.076		118167.310	-55862.598	19918.379	9480.586	50.209	26695.623	144.803	19829.810	1038.552	-95920.666
	(0.009)	(0.094)		(2208.385)	(1573.022)	(607.684)	(323.345)	(4.841)	(8569.459)	(67.956)	(2626.767)	(69.632)	(1040.947)
a1.Manure storage (m^3)	-0.001	0.005	0.001		0.844	-12.152	-1.639	-0.059	-16.419	-0.040	-10.861	-0.586	56.880
	(0.000)	(0.001)	(0.000)		(0.018)	(1.958)	(0.331)	(0.020)	(3.678)	(0.011)	(4.433)	(0.232)	(3.836)
a2.Liquid storage (m^3)	0.005	-0.037	-0.006	209.867		86.166	29.370	0.248	322.308	1.530	113.623	5.115	-475.156
	(0.000)	(0.001)	(0.001)	(7.648)		(4.764)	(1.715)	(0.045)	(105.888)	(0.790)	(20.175)	(0.661)	(13.527)
a5.Digester (m^3)	-0.001	0.006	0.001	-51.812	32.794		-13.498	-0.160	-15.016	-0.210	-18.247	-0.772	67.531
	(0.000)	(0.001)	(0.000)	(4.596)	(3.664)		(1.502)	(0.052)	(6.083)	(0.076)	(9.470)	(0.204)	(6.744)
a6.Digester slurry (m^3)	-0.022	0.204	0.036	-2614.552	1263.591	-494.497		-0.838	-845.666	-3.633	-518.438	-24.928	2429.524
	(0.001)	(0.004)	(0.007)	(68.327)	(53.695)	(29.059)		(0.153)	(265.876)	(1.867)	(70.939)	(2.014)	(44.511)
a7.Biogas storage (m^3)	-0.089	0.826	0.148	-12122.543	5628.695	-2109.732	-957.995		-2733.243	-16.922	-1825.340	-95.922	10150.658
	(0.002)	(0.012)	(0.021)	(262.998)	(181.482)	(91.023)	(36.592)		(801.202)	(6.070)	(223.184)	(6.296)	(148.342)
a9.Clarifier (m^3)	-0.001	0.009	0.002	-134.818	63.297	-23.000	-10.909	-0.077		-0.185	-20.323	-1.068	111.484
	(0.000)	(0.000)	(0.000)	(2.876)	(2.023)	(0.841)	(0.397)	(0.012)		(0.069)	(2.521)	(0.071)	(1.578)
a12.Filtration pond (m^3)	-0.040	0.364	0.066	-5266.877	2451.409	-921.303	-422.924	-3.049	-1188.424		-781.253	-41.895	4459.489
	(0.001)	(0.005)	(0.009)	(110.076)	(77.777)	(39.535)	(15.315)	(0.468)	(349.420)		(96.316)	(2.730)	(64.451)
x1.Number of workers	-0.001	0.009	0.002	-122.550	54.956	-18.790	-9.649	-0.085	-20.120	-0.196		-0.536	97.263
	(0.000)	(0.000)	(0.001)	(2.308)	(1.095)	(0.321)	(0.327)	(0.013)	(2.501)	(0.073)		(0.014)	(0.981)
x2.Capital investment (Million $)	-0.004	0.032	0.006	-465.115	209.214	-72.799	-35.913	-0.262	-87.965	-0.554	-10.079		375.784
	(0.000)	(0.001)	(0.001)	(8.701)	(4.241)	(1.434)	(1.186)	(0.038)	(21.514)	(0.163)	(1.728)		(4.652)
x3.Number of pigs	0.000	-0.000	0.000	1.492	-0.702	0.243	0.113	0.001	0.231	0.002	0.147	0.013	
	(0.000)	(0.000)	(0.000)	(0.017)	(0.012)	(0.004)	(0.003)	(0.000)	(0.028)	(0.001)	(0.026)	(0.001)	
Summer Precipitation (mm)	2.579	-0.024	-0.005	345.691	-160.388	60.346	28.001	0.208	77.945	0.500	52.279	2.749	-292.356
	(0.037)	(0.001)	(0.001)	(7.402)	(5.109)	(2.561)	(1.043)	(0.030)	(22.527)	(0.171)	(6.327)	(0.177)	(4.183)
Summer Temperature (C)	13.818	-0.128	-0.023	1852.541	-859.511	323.394	150.057	1.112	417.704	2.679	280.159	14.732	-1566.725
	(0.200)	(0.002)	(0.003)	(39.669)	(27.377)	(13.721)	(5.591)	(0.162)	(120.722)	(0.915)	(33.904)	(0.952)	(22.413)
Winter Precipitation (mm)	-2.360	0.022	0.004	-316.409	146.802	-55.235	-25.630	-0.190	-71.343	-0.458	-47.851	-2.516	267.593
	(0.034)	(0.001)	(0.001)	(6.776)	(4.676)	(2.344)	(0.955)	(0.028)	(20.619)	(0.157)	(5.790)	(0.163)	(3.828)
Winter Temperature (C)	-8.542	0.079	0.015	-1145.216	531.339	-199.918	-92.764	-0.688	-258.219	-1.657	-173.191	-9.107	968.529
	(0.123)	(0.001)	(0.002)	(24.523)	(16.924)	(8.482)	(3.456)	(0.101)	(74.629)	(0.566)	(20.959)	(0.588)	(13.856)

Note: Standard errors in parentheses below the estimated means are calculated as $SE = s/\sqrt{N}$, where s is the sample standard deviation of the marginal relation across farms and N is the number of observations. Each marginal relation can be tested against the null hypothesis that it equals zero.