Is the Price Right? The Case of the Nutrient Abatement in Chinese Hog Production

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Abstract

Understanding how abatement interacts with production requires a framework that recognizes distinct but interdependent technical processes. This paper applies a Full-Information Maximum Likelihood (FIML) system with endogenous regime switching to three groups of Chinese hog farms using different abatement strategies. Each group operates along two separable frontiers (one for production and one for abatement) linked through the shared land input. Joint estimation identifies the active regime, recovers shadow prices, and measures inefficiency within each technological setting. The separable-frontiers design shows that this structural approach can correct the bias that arises when production and abatement are evaluated within a single-frontier framework. Results reveal wide variation in abatement shadow prices and marginal productivity of pollution abatement across groups, reflecting differences in land use and technology.

Keywords: abatement control, multi-ware technology, land-requirement frontier, shadow price, abatement efficiency

JEL Codes: Q12, Q53, Q57

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1 Introduction

Agricultural production faces a dual challenge: meeting growing food demand while maintaining environmental quality. Livestock systems illustrate this tension. They transform feed, water, and labor into protein but also release nutrient by products that must be managed on farm. The same land that supports production also hosts manure storage, treatment, and reuse. Farms must decide whether to expand abatement facilities, invest in treatment, or intensify production per unit of land. Their choices determine not only farm profitability but also the resulting environmental impact. In Chinese hog production, a range of abatement options exist, from biogas digestion and aerobic-anaerobic treatment to land and fishpond integration. However, their land requirements differ sharply. Understanding how abatement affects land demand and how choice of an abatement-control strategy affects trade offs between output and abatement is central to developing strategies to control emissions.

Economists have long understood that the joint production of desirable and undesirable outputs is a central structural feature of production systems (Ayres and Kneese, 1969; Shephard, 1970; Cropper and Oates, 1992; Førsund, 2021). The material-balance principle, which recognizes that pollution and pollution abatement are embedded within the production process, ensures that abatement activities involve diverting costly inputs from productive activities (Ayres and Kneese, 1969; Coelli et al., 2007; Murty et al., 2012; Abad and Briec, 2019; Dakpo et al., 2016; Murty and Russell, 2020). And numerous empirical studies have shown that incorporating undesirable outputs and material-balance concerns changes measured productivity and efficiency (Ball et al., 2004; Boyd and McClelland, 1999; Pittman, 1983; Hailu and Veeman, 2001; Hoang and Coelli, 2011; Serra et al., 2014). In agriculture, studies show that nutrient abatement entails tangible costs that vary with management, technology, and environmental regulation (Reinhard et al., 1999; Adenuga et al., 2019; Kuhn et al., 2020; Khataza et al., 2017; Doole and Romera, 2014; Chambers et al., 2014).

Our paper studies such problems in the context of Chinese hog production and pollution abatement. Three key issues motivate our approach. First, most empirical studies

treat production and abatement activities as characterized by a single frontier. That ignores the separation of production activities and abatement activities demonstrated by a number of authors (Pethig, 2006; Førsund, 2009; Murty et al., 2012; Førsund, 2018, 2021). That separation of activities also imposes constraints on production activities that are portrayed empirically as distinct, but overlapping, activity-specific frontiers.

Following studies that have employed latent regimes or multi-equation systems for heterogeneous farm technologies (Bokusheva et al., 2012; Kumbhakar and Tsionas, 2016; Areal et al., 2012; Skevas, 2025; O'Donnell, 2010), this paper develops a full-information maximum-likelihood (FIML) model that jointly estimates overlapping "production" and "abatement" frontiers while allowing their parameters and disturbances to interact. The approach retains structural separability while capturing statistical dependence, making it possible to derive shadow prices and substitution patterns consistent with theory.

Second, existing studies often focus on a single production-abatement system that provides little evidence on how distinct abatement portfolios can reshape production tradeoffs. Our data allow us to study three groups of livestock (hog) farms with distinct compliance strategies: land and fishpond integration, aerobic-anaerobic treatment, and biogas systems with polishing. That permits meaningful comparison across abatement portfolios for farms that operate under uniform environmental regulation but with different abatement mechanisms. Empirically evaluating these distinct portfolios reveals how different compliance strategies affect the relationship between production, pollution control, and land use. Thus, our results provide evidence about how structural separability in abatement affects measured efficiency and compliance costs.

Third, empirical measures of environmental efficiency are rarely interpreted in terms of regulatory burden or technological adjustment. We analyze such issues by linking estimated shadow prices and marginal rates of substitution to policies. Doing so provides a spatial view of compliance pressure, showing how environmental regulation interacts with heterogeneous technologies and regional resource scarcity. The results emphasize where abatement requirements are most binding and where innovation or compensation policies may be most effective.

The rest of the paper proceeds as follows. Section 2 introduces the technological representation linking production and abatement. Section 3 describes the data used and variable construction. Section 4 explains the empirical estimation of land based tradeoffs. Section 5 presents the results and policy implications. Section 6 concludes.

2 The Technology

Following Yan and Chambers (2025), we model a joint production and pollution abatement production system. Farms use traditional inputs to produce desirable outputs and their by products using one of N available abatement modules that control emissions of undesirable by products. Technical possibilities are governed by a closed and non-empty *production set*:

$$Z \equiv \{(l, x_y, x_a, y, b, e) : (l, x_y, x_a, y, b, e) \text{ are technically feasible } \},$$

where l denotes land, x_y denotes inputs specific to the production of desirable outputs and by products, x_a denotes inputs specific to abatement activities, y denotes desirable outputs, b denotes by-product production, and e denotes emissions of by products into the environment.

Z is formed as the union of N distinct *production/abatement processes*, Z^n. Each Z^n is closed and non-empty. Superscript "n" distinguishes processes. Thus, $(l^n, x_y^n, x_a^n, y^n, b^n, e^n) \in Z^n$ denotes a feasible activity for the nth process. Therefore,

$$\begin{aligned} Z \equiv \{((l, x_y, x_a, y, b, e) \quad &: \quad (l^n, x_y^n, x_a^n, y^n, b^n, e^n) \in Z^n, n = 1, \ldots, N, \\ (l, x_y, x_a, y, b, e) \quad &= \quad \sum_n (l^n, x_y^n, x_a^n, y^n, b^n, e^n)\}. \end{aligned}$$

2.1 Single-Equation Representation of Production/Abatement Processes

Understanding the interaction of productive and environmental activities is key to our representation. We study livestock (hog) farms that allocate resources to generate desirable outputs, y, and to control (abate) emission of environmentally hazardous by products. This demands specifying a technical process that differs from more conventional single-equation representations.

To illustrate: a conventional single-equation model in our setting would involve specifying for each $n \in \{1, 2, \ldots, N\}$ a *land-requirement function* specific to the nth process defined by

$$\sigma^n(x_y^n, x_a^n, y^n, b^n, e^n) \equiv \inf\{l^n : (l^n, x_y^n, x_a^n, y^n, b^n, e^n) \in Z^n\} \tag{1}$$

that measures the smallest amount of land required to produce (y^n, b^n, e^n) in conjunction with (x_y^n, x_a^n).

We rely on land-requirement functions to specify technical representations because considerable controversy exists over the disposability properties of various inputs and outputs in productive systems involving production of by products. Apart from closedness and non-emptiness of the various Z^n, therefore, the only structural restriction that we impose is free disposability of land in each process. More formally, $(l^n, x_y^n, x_a^n, y^n, b^n, e^n) \in Z^n \Rightarrow (\hat{l}^n, x_y^n, x_a^n, y^n, b^n, e^n) \in Z^n$ for $\hat{l}^n \geq l^n$ for all $n \in \{1, 2, \ldots, N\}$. Free disposability of land ensures that:

$$Z^n = \{(l^n, x_y^n, x_a^n, y^n, b^n, e^n) : l^n \geq \sigma^n(x_y^n, x_a^n, y^n, b^n, e^n)\},$$

so that σ^n exhaustively characterizes Z^n. The land-requirement function links efficiency measurement directly to input requirements. But it does not model how land, y and b interact with activities, x_a, undertaken to control emission of b into the environment, or how y and b interact with land and x_y.

2.2 Multiple-equation Representations

Where there can be no ambiguity, we simplify notation and omit "n" superscripts in what follows.

Following Frisch (1965); Ayres and Kneese (1969); Pethig (2006); Førsund (2009); Murty et al. (2012); Førsund (2018) and Murty and Russell (2020), Yan and Chambers (2025) provide a specification of each process that distinguishes the roles that land, abatement-oriented activities, x_a, production activities, x_y, desirable production, y, undesirable output production, b, and ultimate emissions, e, play in Z as

$$Z = \{(l, x_y, x_a, y, b, e) : l \leq \sigma_y(x_y, b, y), l \leq \sigma_a(x_a, b - e)\}. \tag{2}$$

Here $(b - e)$ defines the level of pollution abatement; $\sigma_y(\cdot)$ represents a land-requirement function specific to production activities where land and x_y interact to produce desirable output, y, and the associated by products, b; and $\sigma_a(\cdot)$ represents a land-requirement function specific to abatement activities that represents how land and x_a interact to remediate undesirable by-products, b, resulting in final emissions of e.

In our empirical application, hog farmers produce meat while emitting nitrogen. Production and controlling emissions are distinct but overlapping activities that compete for the farmer's attention and resources. The specific idea of modeling that competition in terms of two overlapping technical processes traces to the Frisch (1965) notion of "multi-ware production". Førsund (2009) extended that reasoning to encompass "bad outputs", and Murty et al. (2012) and Førsund (2018) provided further developments.

Even though (2) recognizes the overlapping nature of the two processes using two equations, under the assumption that land is freely disposable it still admits a representation in terms of a single-equation land-requirement function.[1] Thus,

$$\sigma_Z(x, y, b, e) = \max\{\sigma_y(x_y, y, b), \sigma_a(x_a, b - e)\}, \tag{3}$$

[1] Let $(l, x, y, b, e) \in Z \Rightarrow (\hat{l}, x, y, b, e) \in Z$ for $\hat{l} \geq l$. Thus, $l \geq \sigma_Z(x, y, b, e) \Rightarrow (l, x, y, b, e) \in Z$. $\Leftarrow$ follows by free disposability because $(\sigma_Z(x, y, b, e), x, y, b, e) \in Z$.

with $\sigma_Z(x, y, b, e) \leq l \Leftrightarrow (l, x, y, b, e) \in Z$.

Both (1) and (3) are single-equation representations. The distinction is that the latter emphasizes that exhaustive knowledge of Z requires knowledge of $\sigma_y(\cdot)$ and $\sigma_a(\cdot)$. That necessitates developing different methods from those needed to estimate (1) to estimate (3). It also requires a more general approach to calculating shadow prices. We address estimation issues in a later section. We now turn to a brief discussion of the use of subdifferential methods for calculating shadow prices.

2.2.1 Measuring Shadow Prices

Representation (3) provides a functional description of the joint production–abatement process that supports the characterization of economically efficient outcomes and shadow prices. The function $\sigma_Z(\cdot)$, however, is not guaranteed to be smooth even if (as in our empirical application) both $\sigma_y(\cdot)$ and $\sigma_a(\cdot)$ are differentiable. Figure 1 illustrates in pollution–land (b, l) netput space, holding other inputs, outputs, and abatement services at $(\bar{x}, \bar{y}, \bar{e})$. Panel 1a plots the two land-requirement functions σ_y and σ_a as functions of b. Each curve is drawn as smooth for visual clarity, and the upper envelope, labelled σ_Z, traces the pointwise maximum of the two processes. That envelope is the multi-ware land-requirement frontier implied by (3). We can partition this envelope into two local regimes: a production regime where σ_y binds and an abatement regime where σ_a binds. Which process governs land use at a given point depends on which frontier delivers the higher land requirement.

The intersection of σ_y and σ_a is highlighted in Panel 1b. At that point both processes bind, and the joint frontier is kinked. Even though each underlying frontier is smooth, their maximum generates a non-differentiable boundary with multiple supporting hyperplanes. Those hyperplanes illustrate the range of real prices that would support an economically efficient outcome at that point of non-differentiability. To accommodate such potential difficulties, we use the notion of a *subdifferential*. For $f(\cdot)$, its subdiffer-

ential correspondence $\partial f(x) \subset \mathbb{R}^N$ is defined:

$$\partial f(x) \equiv \{p \in \mathbb{R}^N : p(v - x) \leq f(v) - f(x),\ \forall v \in \mathbb{R}^N\}. \tag{4}$$

Here p's denote dual (price) vectors (subgradients) that support $f(x)$. If $f(x)$ is convex and smooth, $\partial f(x)$ is the usual gradient. More generally, however, it is not and $\partial f(x)$ accommodates the possibility that multiple hyperplanes may support $f(x)$ if it exhibits a "kink" at x.

Let $q = (q_l, q_x, q_y, q_b, q_e) \equiv (q_l, q_z)$ be a vector of shadow prices for land and $z = (x, y, b, e)$. Economically efficient outcomes are identified with the maximum value a firm can attain across feasible choices for that q. Mathematically, that value for process described by σ_k is given by its *profit function*

$$\sigma_k^*(q) \equiv \sup_z \{q_z z - q_l \sigma_k(z))\}. \tag{5}$$

Each $\sigma_k^*(q)$ is closed, convex, positively homogeneous, and subdifferentiable everywhere on the relative interior of its *effective domain*, dom $\sigma_k^* \equiv \{q : \sigma_k(q) < \infty\}$. Moreover, for $q \in$ ri dom σ_k^*, we have the following general version of the Hotelling-Shephard Lemma (Rockafellar and Wets, 2009; Bertsekas, 2009):

$$\arg\sup\{q_z z - q_l \sigma_k(z)\} = \partial \sigma_k^*(q). \tag{6}$$

Necessary and sufficient conditions for a finite optimum require that[2]

$$\frac{q_z}{q_l} \in \partial \sigma_Z(z). \tag{7}$$

The intuition behind (7) is standard, shadow prices are given by marginal rates of substi-

[2]Let z^o represent an optimizer for (5). By definition,

$$q_z z^o - q_l \sigma_k(z^o) \geq q_z z - q_l \sigma_k(z) \text{ for all } z.$$

Rearranging and using the definition of a subdifferential gives (7). The converse is immediate. Note that unlike first-order conditions for smooth functions, the subdifferential condition is a requirement for a *global* rather than a local optimum.

tution or transformation defined by the slopes of σ_Z. Subdifferentials accommodate the possibility of kinks or discrete differences between binding frontiers.

We note that under our assumption of free disposability of land, only q_l is required to be positive. Other netputs, depending upon the boundary of Z governing the production outcome can have a positive, negative, or zero price. The potential existence of congesting inputs and undesirable by products explains the need to accommodate negative and positive shadow prices. The form of (3) shows that netputs can have zero shadow prices in some instances. Regardless of sign, expression (7) gives the condition required for z to be privately rational.

3 Data

We use micro-level data from China's Ministry of Environmental Protection (MEP) survey of large-scale hog farms conducted between 2012 and 2014. The survey covered all registered farms with more than 500 pigs, which are classified as commercial-scale operations under national livestock regulations. These farms were mandated to install waste-treatment systems under China's 12th Five-Year Plan. The dataset provides detailed information on production, abatement, land use, and pollution outcomes, making it well suited to the joint production and abatement framework. The policy targeted reductions in ammonia nitrogen (NH_3-N) emissions and required farms to adopt approved treatment technologies. Compliance was verified by local authorities, generating a comprehensive record of operational characteristics at the farm level.

Survey coverage and regulatory context. The abatement policy required all large-scale livestock farms to install at least one approved treatment module to control nutrient pollution. Farms could choose from three main technology portfolios depending on site characteristics and management capacity. The first group adopted containment-based systems with land/fishponds integration. The second group employed aerobic-anaerobic treatment systems. The third group developed biogas systems with energy recovery. These groups represent distinct technological portfolios under a single regula-

tory mandate.

Production inputs. The survey records three major production inputs (x): labor, capital (buildings and equipment), and herd size. Herd size serves as a proxy for feed demand, since feed use is determined largely by animal growth requirements at scale. Because direct feed data are unavailable, herd size captures variation in both output production and pollutant generation. This treatment is consistent with prior studies of intensive livestock systems, where herd size closely tracks feed expenditures and nutrient throughput.

Abatement inputs. The survey identifies twelve waste-treatment modules (a), each measured by physical capacity in cubic meters or square kilometers. These include aerobic and anaerobic lagoons, oxidation ponds, clarifiers, digesters, and storage tanks. Each module is treated as a separate abatement input reflecting distinct engineering functions within the treatment process. Although farms often combine modules, pairwise correlations among abatement capacities are modest, confirming that modules can be modeled as separable inputs. The diversity of module combinations across farms reflects the technological breadth of the three groups.

Land input. Land (l) is a shared input that constrains both production and abatement activities. It represents the total footprint required for animal housing, waste storage, and treatment infrastructure. Because abatement systems occupy significant land, especially under containment and lagoon systems, land is directly linked to pollution mitigation capacity. Its role is central in the land requirement frontier estimated in the next section.

Outputs and pollution variables. The desirable output (y) is pork meat, measured in tons. On the environmental side, the data distinguish gross pollutant generation (b), abated pollution (y_a), and residual emissions ($e = b - y_a$). Pollution quantities are reported as NH_3-N in tons, derived from measured or standardized concentration data following MEP guidelines. NH_3-N mass load is the relevant measure for environmental impact and regulatory compliance, capturing total nitrogen discharged into the environment. By distinguishing abated and residual components, the data align precisely with the

by-production framework of joint output and abatement.

Weather data. Each farm is geocoded and matched to seasonal temperature and precipitation data from NOAA grid cells. Average conditions for summer and winter are computed using inverse distance weighting. These variables capture environmental heterogeneity affecting both animal performance and abatement efficiency. Climatic variation is particularly relevant for treatment system design: hotter regions tend to rely on larger aerobic lagoons to maintain stable processing, while colder regions favor anaerobic systems to reduce energy costs.

Summary statistics and group characteristics. Summary statistics by farm group are presented in Table 1. The table shows substantial heterogeneity across technological portfolios. Group 1 farms operate on larger land areas and exhibit lower output density, consistent with the spatial footprint required for containment and reuse systems. Group 2 farms are more capital-intensive and record the highest average output per unit of land, reflecting continuous-flow aerobic-anaerobic treatment systems. Group 3 farms occupy intermediate land scales but invest heavily in energy recovery and polishing facilities, indicating a balanced combination of spatial efficiency and abatement intensity. Variation in abated NH_3-N is wide across all groups, ranging from negligible to over 20 tons per year. This dispersion reflects differences in herd size, abatement infrastructure, and regional climate. The diversity across groups supports the identification of distinct frontiers and allows analysis of how land requirements and shadow prices vary by technology portfolio.

4 Empirical Framework

The theoretical model implies that efficient farms operate under one of two feasible frontiers that jointly determine land use. One frontier reflects the production relationship between inputs, outputs, and gross emissions, while the other captures the abatement relationship between abatement inputs and reduced pollution. Translating that theoretical logic into estimation requires an empirical model that can recover both frontiers jointly

and identify which frontier governs observed behavior at each farm. This section develops such a model and applies it to three groups of hog farms that differ in their abatement configurations. The approach allows us to estimate how land use responds to productive and abatement activities, determine which regime binds, and recover the implied shadow prices and efficiency measures.

We begin by briefly introducing a single-equation benchmark that treats land as an input requirement function of all other netputs. This benchmark remains useful for comparison and for connecting to the more conventional, but it ignores the joint feasibility condition emphasized in the theoretical model. The main analysis adopts a full-information maximum likelihood (FIML) estimator that jointly identifies the production and abatement frontiers and the latent regime that determines which frontier is binding (Orea and Kumbhakar, 2004; Greene, 2005). This specification directly represents the separable frontier structure of the multi-ware model and allows for the observed non-smoothness in land requirements across farms and technologies. The FIML framework also accommodates group-specific abatement portfolios and supports comparisons across three groups of farms and alternative specifications that differ in the inclusion or exclusion of abatement inputs.

4.1 Single-Equation Benchmark

For comparison with conventional approaches, we estimate a single input requirement function that expresses land as a function of all productive and abatement netputs. Specifically,

$$\begin{aligned}\log(l_i+1) = \alpha_0 + \sum_{n=1}^{N} \alpha_n \log(x_{n,i}+1) + \sum_{m=1}^{M} \alpha_m \log(a_{m,i}+1) \\ + \alpha_y \log y_i + \alpha_b \log b_i + \alpha_{ya} \log y_{a,i} + \alpha_w w_i + v_i - u_i,\end{aligned} \tag{8}$$

where l_i is land use, $x_{n,i}$ are production inputs, $a_{m,i}$ are abatement inputs, y_i is meat output, b_i is gross NH_3-N generation, $y_{a,i}$ is abated NH_3-N, and w_i are weather controls. The error term includes a two-part composite: $v_i \sim \mathcal{N}(0, \tau^2)$ for symmetric noise and

$u_i \sim |\mathcal{N}(0, \omega^2)|$ for one-sided inefficiency.

Here the estimated parameters are elasticities that measure proportional changes in land use with respect to each netput. This single-equation model provides an intuitive reference but conflates production and abatement relationships.

Some farms operate at *corner solutions* by deploying a given abatement input equal to zero. To accommodate that possibility in our log-log setting, we transform inputs as $\log(\cdot + 1)$ to accommodate corners, which preserves a coherent logarithmic scale at zero and keeps non-adopters in the sample. This choice maintains continuity of the production and abatement frontiers at the boundary, yields directly comparable elasticities across three groups, and standardizes measurement without altering the underlying economic interpretation.

This transformation introduces a negligible curvature at low input levels, implying that estimated elasticities near zero are locally attenuated relative to the pure log-log form. This bias, however, is intentional and theoretically consistent with the presence of corner solutions. In standard production theory, logarithmic specifications assume interior solutions where all inputs are strictly positive. When some farms do not adopt a particular input, the underlying technology includes both intensive (continuous) and extensive (binary) margins of use. The $\log(\cdot+1)$ form smooths this transition by approximating the intensive margin for adopters while preserving continuity at the boundary for non-adopters. From an empirical standpoint, this adjustment avoids the loss of zero-input observations and maintains a common scale across farms with heterogeneous abatement participation. The elasticity estimated at positive input levels still reflects the proportional responsiveness of land use to marginal input changes, while the local flattening near zero captures the inelastic response at the extensive margin. Thus, the transformation does not distort economic interpretation but embeds a weak separability between adoption and intensity decisions.

4.2 Separable Frontiers and FIML Estimation

The main specification represents the joint technology using two land-requirement frontiers that together define the feasible set. One frontier governs the production process and the other governs abatement. Let $\sigma_y(\cdot)$ and $\sigma_a(\cdot)$ denote the land requirements for production and abatement, respectively. In empirical form, these frontiers are expressed as:

$$\begin{aligned} \log(l_i + 1) &= \sigma_y(z_i; \alpha) + v_{1i} - u_{1i}, \\ \log(l_i + 1) &= \sigma_a(z_i; \beta) + v_{2i} - u_{2i}, \end{aligned} \tag{9}$$

where z_i collects all inputs, outputs, emissions, and weather variables. Each frontier is approximated in log-log form:

$$\begin{aligned} \sigma_y(z_i; \alpha) &= \alpha_0 + \alpha_y \log y_i + \alpha_b \log b_i + \sum_{n=1}^{N} \alpha_n \log(x_{n,i} + 1) + \alpha_w w_i, \\ \sigma_a(z_i; \beta) &= \beta_0 + \beta_{ya} \log y_{a,i} + \sum_{m=1}^{M} \beta_m \log(a_{m,i} + 1) + \beta_w w_i. \end{aligned} \tag{10}$$

The error structure follows the conventional stochastic frontier model with $v_{ji} \sim \mathcal{N}(0, \tau_j^2)$ and $u_{ji} \sim |\mathcal{N}(0, \omega_j^2)|$ for $j = 1, 2$. The production and abatement frontiers jointly define the feasible set of land requirements: $T = (l, z) : l \leq \max[\sigma_y(z), \sigma_a(z)]$. Because only one frontier binds for each observation, we treat the governing regime as latent. The probability that the production frontier binds is modeled as a logistic function of the relative predicted land requirements:

$$P_i = \frac{1}{1 + \exp[-\gamma(\sigma_y(z_i; \alpha) - \sigma_a(z_i; \beta))]}. \tag{11}$$

Each regime contributes to the likelihood through a standard half-normal stochastic frontier density:

$$f_j(\varepsilon) = \frac{2}{\rho_j} \phi\left(\frac{\varepsilon}{\rho_j}\right) \left[1 - \Phi\left(\eta_j \frac{\varepsilon}{\rho_j}\right)\right], \tag{12}$$

where $\rho_j^2 = \tau_j^2 + \omega_j^2$ and $\eta_j = \omega_j / \tau_j$. The overall log-likelihood for observation i is then

given by:

$$\mathscr{L}_i = \log\left[P_i f_1(\log l_i - \sigma_y(z_i;\alpha)) + (1-P_i) f_2(\log l_i - \sigma_a(z_i;\beta))\right]. \tag{13}$$

This specification allows both frontiers to coexist and ensures that regime assignment reflects the tighter land constraint. The FIML estimator jointly identifies the parameters of both frontiers and the regime-selection mechanism, guaranteeing consistent inference on shadow prices, efficiency, and regime probabilities. Relative to two-stage or separated approaches, this joint estimation has two advantages. First, it captures the non-smooth transition where the binding constraint switches between production and abatement. Second, it mitigates bias from unobserved heterogeneity, because farms with systematically different land requirements can be probabilistically allocated to regimes within the same system. The model therefore reflects both observed heterogeneity through weather, scale, and design variables, and unobserved heterogeneity through endogenous regime selection.

5 Results and Implications

This section presents the empirical outcomes from the estimated land-requirement models and interprets their spatial and policy implications. Table 2 reports the estimated coefficients that define the production and abatement frontiers for the three technology groups. Table 3 translates these coefficients into real shadow prices, showing the marginal land cost of each input and output along the active frontier. The three subsections that follow discuss, in order, (i) the estimated frontiers and sign patterns of land elasticities, (ii) the corresponding shadow valuations that quantify spatial tradeoffs, and (iii) policy implications that link the model's geometric interpretation to practical abatement design.

Our estimates do not come from a reduced-form regression. We recover two separable land-requirement frontiers within one system. The FIML likelihood assigns each observation to the binding frontier. Shadow prices and efficiencies are computed on that active surface. The geometry matters for identification and for signs. As noted below, pooling regimes can hide the kink and biases prices, especially for Group 3 farms.

In interpreting these results, it helps to recall that our only disposability assumption involves land. This is done to accommodate known facets of real-world technologies, such as input congestion and lack of disposability. More conventional specifications often maintain that *all* inputs and outputs are freely disposable. One of the more familiar implications of that assumption is that all inputs are substitutes along a fixed isoquant and that all outputs are complementary with land, other factors held fixed. More generally, neither is true. Thus, for example, inputs can be either land-using (complements) or land-saving (substitutes).

5.1 Land-requirement frontiers

Table 2 compares two empirical representations of the same technology: a single-frontier stochastic specification and a FIML system that separates production and abatement frontiers. Both are estimated for three distinct groups of farms, each characterized by its own abatement portfolio. The coefficients measure elasticities of land with respect to production, abatement, and environmental variables. Positive signs indicate land-using effects, while negative signs imply land-saving relationships.

Production outputs (Panel A). Meat output remains the dominant land-using factor across all three groups. They show that expansion of production scale requires proportional growth in land area even after controlling for abatement effort. The effect is strongest among Group 2 farms using aerobic-anaerobic treatment, suggesting that larger production units require proportionally more space for effluent handling and buffer zones. In contrast, farms with integrated recycling systems show smaller elasticities under the FIML model, implying that spatial expansion can be partially absorbed through on-site reuse and dispersion. The NH_3-N coefficient illustrates the opposite dynamic: once abatement is modeled separately, higher emissions per unit of output no longer appear land-using, confirming that signs in single-frontier models reflected statistical rather than technological substitution. This pattern is consistent with more intensive operations generating higher pollution per unit of area (positive marginal rate of substitution between two outputs) and with FIML reducing spurious land-saving signals once abatement is

modeled separately.

Production inputs (Panel A). Labor remains weakly land-using across specifications, representing the managerial and logistical space that accompanies larger operations rather than a direct production requirement. Capital investment shows near-zero or slightly negative effects, suggesting that mechanization substitutes for land only marginally once farms have already installed abatement infrastructure. Herd size, which partly proxies for feed inputs, shows heterogeneity across groups: farms with aerobic-anaerobic treatment exhibit a clear land-saving response, while biogas and recycling systems show neutral or slightly land-using relationships. These differences indicate that herd expansion compresses land only when combined with compact treatment technologies; in other systems, biological processes impose minimum spatial thresholds that prevent further consolidation.

Abated pollution (Panel B). The abated NH_3-N output reveals an important divergence that results from the two estimation methods. In the single-equation Cover, abated NH_3-N appears land-saving in Group 3 and insignificant for other two groups. In the FIML estimation, abated NH_3-N is land-using and precisely estimated, with Group 1 having a relatively large elasticity. This confirms that emission control requires land either for physical infrastructure or for recycling and dispersion. This sign flip reflects correct attribution in the FIML estimation. Once abatement has its own frontier, the land needed for control is no longer misread as production efficiency. Land used for control now can be credited to the abatement frontier, not to production. Group 1 farms rely on land-intensive practices such as integrated fishponds and open storage, so each additional unit of abatement increases a relatively large land footprint. Group 2 and Group 3 achieve similar abatement with a smaller land increment, which suggests greater scope for compact biological or polishing units.

Abatement inputs: storage modules (Panel B). Manure storage remains land-using in all models, but the single-equation frontier overstates its magnitude. Once abatement is estimated separately, FIML recovers smaller positive elasticities, confirming that storage facilities occupy real space within treatment systems. Group 1 becomes in-

significant potentially because fishpond recycling offsets part of the containment burden. For liquid storage, the single-equation results appear negative, implying a spurious land-saving effect. Once the abatement process is isolated in FIML, the coefficients become positive for Groups 1 and 2, while Group 3 remains small and insignificant. The reversal aligns with engineering logic. Under a single frontier, larger storage capacity coincides with higher productivity and thus appears land-saving. FIML correctly reassigns this footprint to the abatement frontier, revealing that open liquid systems expand the land needed for compliance rather than compressing it.

Abatement inputs: biological and chemical treatment systems (Panel B). The following abatement inputs, aerobic lagoon, anaerobic lagoon, digester slurry, integrated fishpond, clarifier, and oxidation pond, define the treatment backbone of farms adopting engineered aerobic-anaerobic or polishing systems. They show consistent sign patterns across both estimation methods, suggesting stable spatial implications rooted in system design. Aerobic lagoons are land-saving under all specifications because they operate at higher loading rates and shorter retention times, allowing treatment in compact basins. In contrast, anaerobic lagoons remain land-using in both frameworks since slow decomposition under low-oxygen conditions demands deeper ponds, longer retention, and larger footprints, which increase both volume and surface requirements. Digester slurry and integrated ponds, both adopted by Group 1, show positive coefficients, indicating land usages once recycling or reuse becomes fully functional. Clarifiers are also consistently land-using because sedimentation and controlled flow require fixed basin space. Although the single-equation model sometimes dilutes their significance due to correlation with other components, FIML clarifies their independent contribution by isolating abatement from production. Oxidation ponds, too, are land-using throughout, as aeration and stabilization depend on wide surface exposure. Together, these results show that for systems relying on fundamental biological and chemical processes, land requirements are intrinsic and consistently identified across econometric frameworks.

Abatement inputs: energy recovery and chemical systems (Panel B). Similar to the liquid storage input, the remaining inputs: digester, biogas storage, hydrochlo-

ric acid pond, and filtration pond, each display some sign reversals between estimation frameworks, indicating how FIML corrects misattributed land effects for certain groups. The digester turns from land-saving under the single-equation model to land-using under FIML for Group 3, which properly assigns the physical space of digestion tanks and heating units to abatement rather than to production efficiency. The hydrochloric acid pond shifts from insignificant to strongly land-using, consistent with its design as a lined, segregated chemical treatment basin that requires safety spacing. The filtration pond, conversely, turns from positive to negative once abatement is isolated, reflecting its compact polishing function that replaces larger clarifier or oxidation units.

Under FIML estimation, these sign corrections clarify the spatial logic of energy recovery and polishing systems. When abatement is treated as part of production, land devoted to treatment infrastructure is absorbed into measured efficiency, producing misleading signs. Once the two frontiers are estimated jointly, each module's spatial role becomes visible: digestion and acid neutralization expand land needs through dedicated containment, while gas storage and filtration reduce total footprint by stabilizing and consolidating treatment flows. The FIML framework therefore reconciles econometric outcomes with engineering design, showing that true land savings emerge from integrated systems that smooth and polish treatment rather than from core energy recovery or chemical conversion units.

Weather variables (Panels A and B). Seasonal weather effects are consistent across frameworks but reallocated once production and abatement frontiers are separated. In both systems, summer temperature and precipitation are land-saving, while winter conditions are land-using, reflecting biological versus storage demands. Under the single-equation frontier, these effects appear in production because all weather variation is pooled there. FIML corrects this by showing that most of the response originates in abatement regimes, where temperature and moisture directly affect retention, decomposition, and treatment stability. Group 1 farms show the strongest seasonal contrasts. Their open recycling and fishpond systems expand during hot summers. For all groups, land saving occurs in abatement regimes in mild winters, inverting the typical signs. FIML

captures this heterogeneity by assigning the weather-driven spatial variation to the abatement frontier rather than to production scale. This insight matters for adaptation policy. It implies that mitigating land pressure under climate variability depends on resilient abatement infrastructure rather than curbing production itself.

Together, the FIML separation provide a coherent spatial logic of environmental technology: land efficiency depends not only on production scale but also on the configuration and integration of abatement systems within each portfolio.

5.2 Shadow prices among groups

Shadow prices quantify the marginal value of land with respect to each input and output along the estimated frontiers. They represent the implicit land cost of expanding production or abatement by one unit while maintaining efficiency. Table 3 reports these real shadow prices, expressed directly in land units, showing the marginal land cost or saving associated with a one-unit change in each variable. Specifically, we define the shadow price of variable z_j (one element of the vector z collecting all inputs, outputs, emissions, and weather variables) as the marginal land requirement on the active frontier,

$$SP_j(z) = \frac{\partial \sigma_r(z)}{\partial \log z_j} \cdot \frac{l_r(z)}{z_j}, \quad r \in \{y, a\},$$

measured in m^2 per unit of z_j. Because the FIML model embeds two regimes, the production and abatement frontiers, shadow prices are computed for each while holding the other regime's inactive. This setup generates the characteristic "kink" and switching frontiers. Some shadow prices are zero when the inactive regime does not bind. These zeros, omitted from Table 3 but noted in the panel titles, mark the transition points that connect production and abatement. Recognizing this kink is essential in a joint-production context because it links technological feasibility to the valuation of land. For comparison, we list the single-frontier shadow prices calculated in the same way but ignore the structure and pool both regimes.

While Table 2 described how inputs and outputs influence land requirements, Table

3 reveals the corresponding opportunity value of land implied by those effects. Because the estimates are derived directly from the real-valued FIML frontiers, the shadow prices are measured in physical land units rather than monetary terms. Their magnitudes thus represent spatial costs, allowing direct comparison of land efficiency across groups.

Production outputs (Panel A). The shadow prices of meat output are generally larger under the FIML system than under the single frontier for Groups 2 and 3, indicating that once abatement is modeled separately, the true shadow burden of production becomes apparent. The higher land cost in Group 2 confirms that aerobic-anaerobic systems, although compact in engineering design, still require proportional expansion of treatment capacity as output grows, so their marginal land shadow cost rises with scale. Group 1 exhibits the smallest shadow price, consistent with its integrated layout that spreads additional production over preexisting land uses and shared recycling space.

Unabated NH_3-N consistently carries a negative shadow price in FIML, which means that not abating pollution saves land. The saving comes from avoiding containment, storage, and polishing area. Group 3 shows the most negative value, reflecting limited spare buffer capacity in its compact configuration. From an environmental perspective, this negative price signals an externalization: land is saved on the farm while the environmental burden rises off the farm. The absolute values exceed those for meat output because a marginal ton of NH_3-N relaxes multiple abatement land requirements at once, so the land response is large. Under FIML the absolute magnitudes shrink compared with the single frontier because the model reallocates abatement footprint to the abatement frontier, removing spurious land-using signals from the production block.

Production inputs (Panel A). Among production factors, labor retains positive but moderate shadow prices, highest in Group 3 and lowest in Group 1. This ordering mirrors the broader spatial footprint of labor-intensive handling in the complex biogas systems. Capital investment and herd size change roles once abatement are modeled separately, yet the numbers remained small and insignificant.

Abated pollution (Panel B). Panel B presents the shadow prices when abatement activities bind and production variables' shadow prices are set to zero. The most notable

observation concerns the shadow price of abated NH_3-N, which turns positive and reduces in magnitude under FIML. The large shadow value indicates that emission control imposes real spatial requirements once disentangled from production efficiency. Group 1 exhibits the highest shadow price among all groups, consistent with the land-intensive nature of its recycling-based abatement. Groups 2 and 3 display smaller shadow prices, reflecting the compact design of engineered and biogas systems. The ranking of magnitudes aligns with system architecture: land-based recycling generates the strongest spatial burden, while closed treatment and recovery systems economize on land.

Figure 2 illustrates the spatial distribution of the county-level shadow prices of abated NH_3-N across mainland China. The three subfigures (a) to (c) correspond to the three abatement groups and reveal marked heterogeneity in the spatial burden of emission control. High shadow values cluster in the south and parts of the northeast, where livestock density and land scarcity potentially raise the marginal value of abatement. In contrast, counties in the central and western regions show relatively lower shadow prices, reflecting more abundant land or less intensive operations.

Abatement inputs: storage modules (Panel B). The shadow prices of manure storage remain positive across all three groups under the FIML estimation, confirming that containment continues to impose land requirements even when abatement and production frontiers are separated. However, their magnitudes are substantially smaller than in the single-frontier estimates, suggesting that part of the land burden previously attributed to storage is now absorbed by production activities. For liquid storage, the reallocation is more pronounced. Groups 1-2 move from negative to positive shadow prices once the abatement frontier is identified, revealing that these facilities expand spatial demand potentially higher than the manure storage. Group 3 remains close to zero, reflecting compact or closed-loop designs.

Abatement inputs: biological and chemical treatment systems (Panel B). The shadow prices for core treatment units reveal how process design governs spatial efficiency across systems. Aerobic lagoons exhibit negative shadow prices, showing that rapid oxidation and short retention allow biological treatment in compact basins. Anaero-

bic lagoons, by contrast, display positive shadow prices in both cases, with a smaller value in FIML than in the single-frontier model. This indicates that when anaerobic treatment estimated with a separable abatement frontier, its land intensity can be mitigated.

Shadow prices for the reuse and recycling modules reveal the spatial implications of closing nutrient loops on farm sites. For digester slurry and integrated fishponds, both exhibit a relatively large positive shadow price, indicating that reuse becomes spatially costly once the material must be processed through land or pond integrations. Extra lands are required to host aquaculture infrastructure. Space is needed not only for open storage but also for waste treatment. The fishpond's shadow value has a large magnitude, potentially indicating its dual role in abatement and production.

Clarifiers also maintain positive shadow prices, reflecting that even when integrated with biogas recovery, sedimentation and controlled flow require fixed space commitments. Oxidation ponds similarly show positive shadow prices with their magnitude exceeds that of aerobic lagoons, consistent with their reliance on extended surface exposure for stabilization.

Abatement inputs: energy recovery and chemical systems (Panel B). The shadow prices for advanced treatment and recovery modules illustrate how specialized engineering alters spatial requirements within abatement systems. Digesters remain land saving in Group 1 but switch the sign in FIML estimation for Group 3, showing that when energy recovery units are central to a farm's configuration, the space devoted to reactors and heating equipment becomes a more significant component of total land demand. Compared to digesters, biogas storage shows the opposite case in the single-equation frontier model but presents a similar pattern in FIML. Its negative shadow price in Group 1 potentially signifies that stabilized gas retention alleviates pressure on other containment facilities.

Hydrochloric acid ponds experience a sign switch in FIML. The results appear more consistent with the safety and isolation standards required for chemical neutralization. Filtration ponds, by comparison, record a negative shadow price in FIML, potentially indicating compact polishing roles that replace more space-intensive clarifiers or oxidation

ponds.

Overall, the FIML system is essential for identifying the shadow values, especially for abatement inputs, in a joint production system. By estimating both frontiers simultaneously, it assigns shadow prices to the regime that truly binds, recognizing the overlapping frontiers that govern production relations. Single-equation models collapse the kink into a smooth surface, producing shadow prices that blend production and abatement effects and lose economic meaning. By averaging over regimes, the single equation assigns nonzero prices to inactive variables, reverses plausible signs, and exaggerates magnitudes for modules whose roles depend on the binding frontier.

5.3 "Is the price of abatement right?"

The central lesson for the policy analyst is simple. Prices inferred from a single frontier are likely inappropriate to policy design. They conflate production and abatement constraints and often assign nonzero shadow values to inactive regimes. Our estimates instead anchor prices on the regime that actually binds and respect the kinked shape of the joint frontier.

The estimated model can quantify land requirements and output trade-offs under alternative abatement portfolios. The results in Figure 2, Panels (d) to (f) report the marginal product of pollution abated (MPPA), defined as the negative ratio of shadow prices for abated pollution and meat output ($-\text{SP}_{y_a}/\text{SP}_y$). This index represents the trade-off between productive output and abatement under a constant land constraint, effectively measuring how much abatement can be achieved per unit of forgone output when land is held fixed.

The MPPA estimates are uniformly negative, confirming that pollution abatement and productive output compete for the same land resource. *In other words, abatement is costly.* Even for the smallest tradeoff group (Group 2), the mean MPPA of about 10 implies that abating one ton of NH_3-N entails an expected loss of roughly ten tons of meat when total land remains fixed. This ratio soars to extreme levels for Group 1. This pattern reflects the disproportionate land pressure that arises when farms depend on a narrow set of containment and storage technologies to absorb wastes through land or

fishponds integrations. Groups 2 and 3, whose abatement systems integrate treatment or recycling, maintain more moderate trade-offs, indicating that diversified portfolios reduce the land-output competition intensity. These results align with the theoretical prediction that the joint frontier's kink steepens as abatement flexibility declines, raising the marginal cost of pollution reduction in terms of forgone production.

The maps in Panels (d)-(f) of Figure 2 reveal substantial spatial dispersion in the marginal product of pollution abated (MPPA) across counties. Even within each group, MPPA values differ considerably, indicating that the trade-off between abatement and output is not uniform across space. The absence of clear geographic clustering suggests that location-specific production environments matter more rather than broad regional trends. This scattered pattern shows the challenge of designing uniform land-use or abatement policies, as the marginal value of pollution control appears highly uneven even among farms operating under similar technological conditions.

Appendix Figure A1 plots shadow values and MPPA from the single-frontier benchmark using the same county grid. Comparing Figure 2 to Figure A1 shows why structure matters. For Group 3, the single-frontier maps compress the level of abatement shadow prices and flatten MPPA. In many counties the benchmark even suggests benign or favorable tradeoffs that look like "free" abatement. The FIML maps correct this by locating the active abatement frontier and restoring the land burden at the margin.

These results yield two main policy implications. First, the structure shows that efficiency-oriented environmental policy should avoid imposing a single abatement module across heterogeneous farms. Instead, policy could promote menu-based combinations that let farms select complementary technologies suited to their land and capital constraints. If farms operate within a broader selection, abatement could be achieved with lower incremental land cost. Second, the results suggest that large pollution reductions can occur without expanding total land if abatement module composition is rebalanced. This finding indicates that regulatory programs encouraging integrated or mixed portfolios can achieve greater abatement per unit of land, lowering the opportunity cost of environmental compliance in terms of forgone production.

From a broader perspective, our production-abatement FIML framework provides a quantitative basis for designing incentive-compatible abatement policies. By revealing the expected efficiency gains from each farm group's technological menu, it suggests that heterogeneity in abatement access should be viewed as a policy instrument for cost-effective regulation rather than a sign of inefficiency. Allowing farms to adopt complementary technologies within a broader selection of inputs can reduce potential spatial distortions and aligns land-use efficiency with environmental goals. The model thus formalizes the link between technological diversity, land efficiency, and pollution control, providing a coherent foundation for evaluating instruments that encourage multi-module adoption in agricultural systems.

6 Concluding Remarks

This paper develops and applies a multi-ware framework to analyze the joint land requirements of production and abatement in Chinese hog farming. The framework represents the technology as the intersection of two input requirement frontiers, one governing production and the other abatement, whose joint envelope defines the feasible land set. Each frontier captures a distinct constraint, and their intersection generates a kinked surface where the binding regime switches endogenously. This structure enables the measurement of efficiency and shadow prices even when the frontier is non-smooth.

The empirical results show that separating production and abatement frontiers yields clearer and more consistent measures of efficiency than single-frontier models. Farms relying on land-intensive recycling and integration face higher marginal land burdens, while those adopting compact aerobic-anaerobic or biogas systems achieve greater spatial efficiency. The Full-Information Maximum Likelihood (FIML) estimation approach distinguishes the active regime at each farm and captures how regulatory pressure and technology design jointly shape land use. These results confirm that treating production and abatement as a single process conceals important heterogeneity in compliance behavior. They also show that shadow prices taken from a single frontier can give a false sense

of accuracy. When production and abatement share land, the active constraint shifts with scale, season, and technology. A smooth frontier cannot track this movement. As a result, prices that look precise on a pooled surface can misrepresent both the sign and the size of the true marginal land burden faced by farms.

The findings carry three applied economic implications. First, the findings suggest that environmental regulation should move away from uniform mandates toward flexible, menu-based abatement standards. Allowing farms to combine multiple modules can reduce the marginal land cost of pollution control while preserving output efficiency. Regulatory designs that permit technological mixing, such as integrating compact polishing or chemical treatment units within existing systems, can achieve comparable abatement levels with smaller land footprints.

Second, the results emphasize that land scarcity is itself an environmental constraint. The estimated shadow prices quantify the opportunity cost of allocating land to abatement rather than production, revealing that environmental compliance competes directly with food supply for space. Recognizing this trade-off is critical for setting regionally differentiated standards. In areas where land pressure is high, policies should prioritize compact abatement technologies and provide targeted support for their adoption.

Third, the analysis shows that seasonal and climatic conditions shape the effective land burden of abatement. Hot summers and cold winters alter treatment efficiency and storage demands, shifting which frontier binds. This pattern implies that adaptive or season-sensitive regulation could maintain both environmental and spatial efficiency. Support for temperature-stable treatment units or integrated systems can buffer farms against these cyclical constraints.

Together, these results underscore the value of modeling production and abatement as interconnected but separable processes. Recognizing which frontier binds and why allows policymakers to design incentive-compatible regulations that align environmental and spatial efficiency. The separable-frontier framework provides a practical tool for measuring compliance costs, identifying binding constraints, and evaluating the land implications of technology choice. More broadly, the approach can be extended to other

resource-intensive sectors where production and environmental management share limited inputs. It offers a transparent foundation for policies that internalize land as both a productive asset and an environmental constraint, central to achieving sustainable growth in agriculture.

In sum, *the price is not right* when production and abatement are evaluated on a single frontier. Correctly pricing the shared input requires recognizing which regime binds and why. The multi-ware approach presented here establishes that foundation, offering both a theoretical and empirical path toward environmental policy that internalizes land as a scarce but indispensable factor in agricultural abatement.

Table 1: Summary Statistics for Commercial Hog Farms by Group in China (2012-2014)

	Group 1		Group 2		Group 3	
Variable	Level	Log	Level	Log	Level	Log
y-Meat output (ton)	467.978	5.542	605.919	5.739	470.630	5.556
	(948.859)	(0.952)	(1058.333)	(1.048)	(821.052)	(0.958)
b-NH_3-N (ton)	1.523	0.043	1.832	0.180	1.597	0.110
	(3.466)	(0.902)	(3.459)	(0.970)	(2.934)	(0.917)
y_a-Abated NH_3-N (ton)	5.371	0.984	7.153	1.216	5.449	1.029
	(12.134)	(0.970)	(13.480)	(1.077)	(10.194)	(0.965)
l-Farm Area (1000 m^2)	3.413	0.717	1.510	0.525	2.042	0.507
	(44.981)	(0.674)	(5.078)	(0.688)	(42.190)	(0.609)
a_1-Manure storage (m^3)	673.908	5.332	796.414	5.622	738.224	5.480
	(1909.795)	(1.936)	(2026.338)	(1.646)	(1878.790)	(1.849)
a_2-Liquid storage (m^3)	851.396	4.861	1006.751	4.953	758.921	4.530
	(4800.690)	(2.690)	(4141.576)	(2.778)	(3375.834)	(2.877)
a_3-Aerobic lagoon (m^3)	–	–	331.888	2.483	–	–
	–	–	(3027.276)	(2.925)	–	–
a_4-Anaerobic lagoon (m^3)	–	–	459.753	4.032	–	–
	–	–	(1114.442)	(2.839)	–	–
a_5-Digester (m^3)	426.688	3.362	–	–	575.014	5.066
	(4128.510)	(2.816)	–	–	(3277.150)	(1.950)
a_6-Digester slurry (m^3)	146.245	0.929	–	–	–	–
	(660.167)	(2.211)	–	–	–	–
a_7-Biogas storage (m^3)	6.256	0.099	–	–	6.000	0.086
	(122.383)	(0.700)	–	–	(128.020)	(0.663)
a_8-Hydrochloric acid pond (m^3)	–	–	20.256	0.164	–	–
	–	–	(485.453)	(0.928)	–	–
a_9-Clarifier (m^3)	–	–	–	–	34.757	0.214
	–	–	–	–	(1296.333)	(1.063)
a_{10}-Integrated fishpond (km^2)	1.439	0.194	–	–	–	–
	(25.277)	(0.606)	–	–	–	–
a_{11}-Oxidation pond (m^3)	–	–	486.024	0.895	–	–
	–	–	(4520.175)	(2.388)	–	–
a_{12}-Filtration pond (m^3)	–	–	–	–	5.497	0.050
	–	–	–	–	(275.132)	(0.504)
x_1-Number of workers	29.112	0.267	45.935	0.377	21.783	0.246
	(384.495)	(0.978)	(525.052)	(1.192)	(324.589)	(0.919)
x_2-Capital investment (Million $)	7.926	0.803	9.903	0.971	6.948	0.817
	(72.801)	(1.008)	(88.804)	(1.078)	(63.447)	(1.009)
x_3-Number of pigs	3829.853	7.644	4991.952	7.859	3914.451	7.693
	(8137.798)	(0.948)	(8821.301)	(1.040)	(6841.450)	(0.948)
Summer Precipitation (mm)	192.211		186.320		186.483	
	(88.443)		(84.355)		(83.663)	
Winter Precipitation (mm)	107.298		97.151		101.156	
	(83.668)		(87.182)		(82.676)	
Summer Temperature (C)	25.857		25.430		25.747	
	(2.545)		(2.526)		(2.510)	
Winter Temperature (C)	12.803		12.648		12.892	
	(4.625)		(3.725)		(3.995)	
Observation	5286		6213		16015	

Note: Entries report means; standard deviations are shown in parentheses on the row below each mean.

Table 2: Land Requirement: Single-Frontier (by group) vs. FIML Separable Frontier (by group). (China Commercial Hog Farms, 2012–2014)

	Single-Frontier SFA			**FIML Separable Frontier**		
	G1	G2	G3	G1	G2	G3
Dependent variable: Farm Area (1000 m^2)						
Panel A. Production frontier						
Meat output (ton)	0.703***	1.025***	0.196***	0.383***	1.651***	0.439***
	(0.066)	(0.053)	(0.035)	(0.035)	(0.085)	(0.051)
NH_3-N (ton)	0.172	0.00898	-0.300***	-0.0609***	-0.0337	-0.105***
	(0.096)	(0.069)	(0.061)	(0.018)	(0.028)	(0.024)
Number of workers	0.0335***	0.00423	0.0155**	0.00279	0.00559	0.0162*
	(0.008)	(0.007)	(0.005)	(0.005)	(0.010)	(0.007)
Capital investment (Million $)	-0.00121	-0.00309	0.000301	-0.000199	0.00707	-0.00515
	(0.008)	(0.007)	(0.005)	(0.004)	(0.011)	(0.007)
Number of pigs	-0.854	-0.902**	1.085***	0.100*	-1.099***	0.0116
	(0.452)	(0.320)	(0.284)	(0.043)	(0.089)	(0.059)
Summer Precipitation (mm)	-0.00135***	-0.00262***	-0.00143***	-0.000467***	-0.00472***	-0.00209***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Summer Temperature (C)	-0.00237	-0.0250***	-0.00484	-0.000617	-0.0393***	-0.00274
	(0.004)	(0.004)	(0.003)	(0.002)	(0.007)	(0.004)
Winter Precipitation (mm)	0.000925***	0.00247***	0.00120***	0.000294***	0.00332***	0.00141***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Winter Temperature (C)	0.00547*	0.0165***	0.000419	0.00281*	0.0293***	0.00203
	(0.003)	(0.003)	(0.002)	(0.001)	(0.004)	(0.003)
Constant	1.942	0.689	-8.413***	-2.620***	-1.399***	-2.814***
	(3.061)	(2.171)	(1.933)	(0.187)	(0.345)	(0.255)
Panel B. Abatement frontier (SFA shows duplicated weather from Panel A for comparability)						
Abated NH_3-N (ton)	0.415	0.147	-0.687**	0.167***	0.0977***	0.0883***
	(0.352)	(0.249)	(0.222)	(0.020)	(0.002)	(0.001)
a_1 Manure storage (m^3)	0.0254***	0.00835	0.0165***	0.00134	0.00759***	0.00276***
	(0.005)	(0.005)	(0.003)	(0.005)	(0.001)	(0.001)
a_2 Liquid storage (m^3)	-0.00910**	-0.00844**	-0.00754***	0.00380	0.00357***	-0.00000181
	(0.003)	(0.003)	(0.002)	(0.003)	(0.001)	(0.000)
a_3 Aerobic lagoon (m^3)	–	-0.00202	–	–	-0.00416***	–
	–	(0.003)	–	–	(0.001)	–
a_4 Anaerobic lagoon (m^3)	–	0.0488***	–	–	0.00387***	–
	–	(0.003)	–	–	(0.001)	–
a_5 Digester (m^3)	-0.00830**	–	-0.0418***	-0.00295	–	0.00614***
	(0.003)	–	(0.002)	(0.003)	–	(0.001)
a_6 Digester slurry (m^3)	0.00307	–	–	0.00447	–	–
	(0.003)	–	–	(0.003)	–	–
a_7 Biogas storage (m^3)	0.0153	–	0.0666***	-0.0201	–	0.000922
	(0.010)	–	(0.006)	(0.011)	–	(0.002)
a_8 Hydrochloric acid pond (m^3)	–	-0.00317	–	–	0.00633***	–
	–	(0.007)	–	–	(0.002)	–
a_9 Clarifier (m^3)	–	–	0.00735	–	–	0.00319***
	–	–	(0.004)	–	–	(0.001)
a_{10} Integrated fishpond (km^2)	0.0280*	–	–	0.0606***	–	–
	(0.012)	–	–	(0.012)	–	–
a_{11} Oxidation pond (m^3)	–	0.00708*	–	–	0.00287***	–
	–	(0.003)	–	–	(0.001)	–
a_{12} Filtration pond (m^3)	–	–	0.0206*	–	–	-0.00314
	–	–	(0.008)	–	–	(0.002)
Summer Precipitation (mm)	-0.00135***	-0.00262***	-0.00143***	-0.000197	-0.0000681*	0.0000700***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Summer Temperature (C)	-0.00237	-0.0250***	-0.00484	0.0230***	-0.00121	-0.00112*
	(0.004)	(0.004)	(0.003)	(0.005)	(0.001)	(0.000)
Winter Precipitation (mm)	0.000925***	0.00247***	0.00120***	0.000249	0.000294***	0.0000315*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Winter Temperature (C)	0.00547*	0.0165***	0.000419	-0.0136***	-0.000787	-0.00287***
	(0.003)	(0.003)	(0.002)	(0.003)	(0.001)	(0.000)
Constant	1.942	0.689	-8.413***	0.337**	0.137***	0.110***
	(3.061)	(2.171)	(1.933)	(0.113)	(0.017)	(0.011)
Panel C. Distributional and Inefficiency Parameters (FIML only)						
γ	–	–	–	-8.520***	3.441***	0.743***
				(0.709)	(0.156)	(0.037)
σ_{u1}	–	–	–	-1.307***	-19.68	-28.42***
				(0.030)	(538.235)	(0.000)
σ_{v1}	–	–	–	-2.270***	-0.169***	-0.537***
				(0.040)	(0.017)	(0.008)
σ_{u2}	–	–	–	-29.76***	-3.535***	-4.548***
				(0.000)	(0.145)	(0.354)
σ_{v2}	–	–	–	-0.0371	-2.847***	-3.012***
				(0.023)	(0.022)	(0.018)
Observations	5,286	6,213	16,015	5,286	6,213	16,015

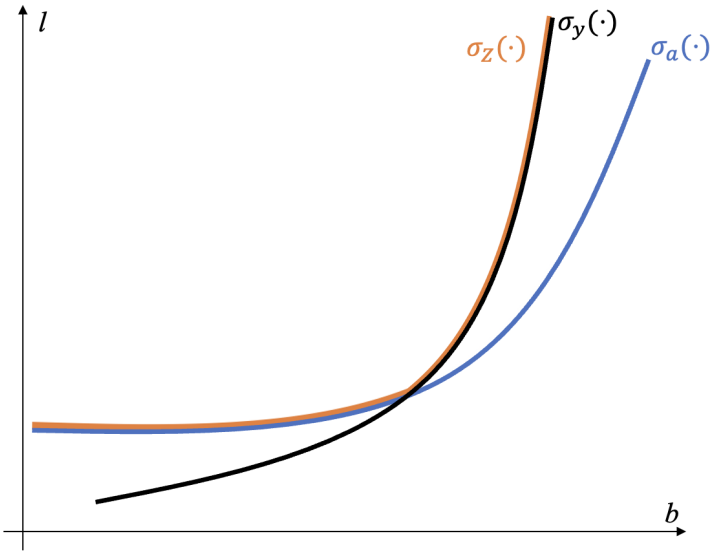
Notes: All specifications are log-log with errors following a half normal distribution. Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Weather terms and constant in the Panel B are duplicated for SFA from Panel A for comparison.

Table 3: Real Shadow Prices (in land) by Group: Single-Frontier vs. FIML Separable Frontier. (China Commercial Hog Farms, 2012–2014)

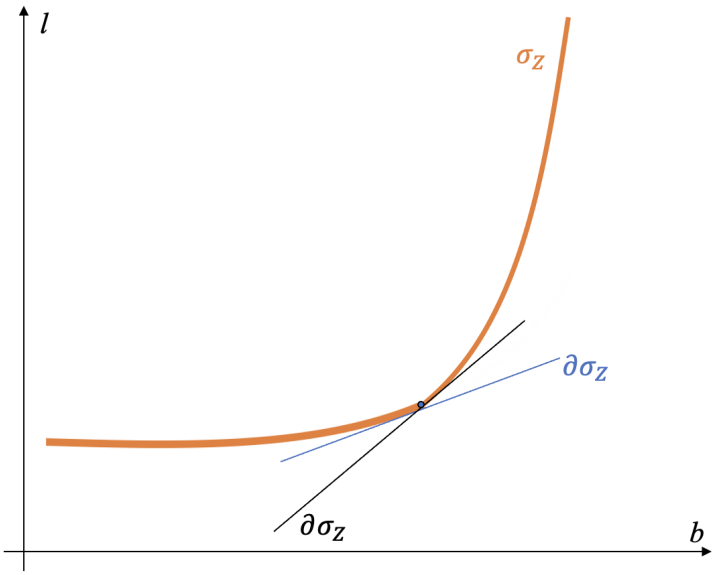
	Single-Frontier SFA			**FIML Separable Frontier**		
	G1	G2	G3	G1	G2	G3
Panel A. Production-block variables (SFA single-frontier) / Production-regime shadow prices (FIML)						
y Meat output (ton)	0.00645	0.00722	0.00158	0.00178	0.00783	0.00426
	(0.00004)	(0.00006)	(0.00001)	(0.00001)	(0.00011)	(0.00002)
b NH_3-N (unabated, ton)	0.38733	0.01672	-0.56135	-0.09295	-0.05506	-0.24051
	(0.00249)	(0.00016)	(0.00273)	(0.00143)	(0.00087)	(0.00145)
x_1 Number of workers	0.06549	0.00675	0.02405	0.00913	0.01162	0.03232
	(0.00044)	(0.00005)	(0.00007)	(0.00011)	(0.00015)	(0.00013)
x_2 Capital investment (Million \$)	-0.00154	-0.00293	0.00030	-0.00038	0.00828	-0.00662
	(0.00001)	(0.00003)	(0.00000)	(0.00001)	(0.00013)	(0.00004)
x_3 Number of pigs	-0.00096	-0.00078	0.00104	0.00006	-0.00069	0.00001
	(0.00001)	(0.00001)	(0.00000)	(0.00000)	(0.00001)	(0.00000)
Summer precipitation (mm)	-0.00135	-0.00262	-0.00143	-0.00215	-0.01307	-0.00473
	(0.00000)	(0.00000)	(0.00000)	(0.00002)	(0.00014)	(0.00002)
Summer temperature (C)	-0.00237	-0.02498	-0.00484	-0.00285	-0.10885	-0.00623
	(0.00000)	(0.00000)	(0.00000)	(0.00002)	(0.00120)	(0.00002)
Winter precipitation (mm)	0.00092	0.00247	0.00120	0.00136	0.00920	0.00320
	(0.00000)	(0.00000)	(0.00000)	(0.00001)	(0.00010)	(0.00001)
Winter temperature (C)	0.00547	0.01654	0.00042	0.01296	0.08125	0.00462
	(0.00000)	(0.00000)	(0.00000)	(0.00010)	(0.00090)	(0.00002)
Panel B. Abatement-block variables (SFA single-frontier; weather duplicated from Panel A)						
y_a Abated NH_3-N (ton)	0.36846	0.09849	-0.51840	0.29075	0.07276	0.05932
	(0.00230)	(0.00088)	(0.00247)	(0.00228)	(0.00076)	(0.00090)
a_1 Manure storage (m^3)	0.00422	0.00077	0.00197	0.00032	0.00031	0.00024
	(0.00021)	(0.00005)	(0.00006)	(0.00002)	(0.00002)	(0.00002)
a_2 Liquid storage (m^3)	-0.00457	-0.00413	-0.00363	0.00195	0.00070	-0.00000
	(0.00014)	(0.00011)	(0.00005)	(0.00006)	(0.00003)	(0.00000)
a_3 Aerobic lagoon (m^3)		-0.00213			-0.00306	
		(0.00003)			(0.00004)	
a_4 Anaerobic lagoon (m^3)		0.02338			0.00139	
		(0.00047)			(0.00003)	
a_5 Digester (m^3)	-0.00779		-0.00926	-0.00322		0.00019
	(0.00016)		(0.00022)	(0.00006)		(0.00002)
a_6 Digester slurry (m^3)	0.00595			0.01069		
	(0.00006)			(0.00007)		
a_7 Biogas storage (m^3)	0.03410		0.11340	-0.05647		0.00106
	(0.00025)		(0.00031)	(0.00015)		(0.00000)
a_8 Hydrochloric acid pond (m^3)		-0.00580			0.00745	
		(0.00004)			(0.00002)	
a_9 Clarifier (m^3)			0.01226			0.00357
			(0.00004)			(0.00002)
a_{10} Integrated fishpond (km^2)	0.05704			0.15422		
	(0.00046)			(0.00066)		
a_{11} Oxidation pond (m^3)		0.01167			0.00295	
		(0.00010)			(0.00002)	
a_{12} Filtration pond (m^3)			0.03556			-0.00366
			(0.00010)			(0.00001)
Summer precipitation (mm)	-0.00135	-0.00262	-0.00143	-0.00056	-0.00008	0.00008
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Summer temperature (C)	-0.00237	-0.02498	-0.00484	0.06519	-0.00146	-0.00131
	(0.00000)	(0.00000)	(0.00000)	(0.00014)	(0.00000)	(0.00000)
Winter precipitation (mm)	0.00092	0.00247	0.00120	0.00071	0.00035	0.00004
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
Winter temperature (C)	0.00547	0.01654	0.00042	-0.03867	-0.00095	-0.00336
	(0.00000)	(0.00000)	(0.00000)	(0.00008)	(0.00000)	(0.00001)

Notes: Real shadow prices use coefficients from Table 2 via $SP_{z,i} = \hat{\beta}_z \cdot (l_i/z_i)$. Entries are means; standard errors shown in parentheses computed as $SD/\sqrt{n}$. For SFA, n is group sample size (G1=6,095; G2=7,034; G3=18,459). For FIML G1, production-regime n = 1,863 and abatement-regime n = 4,232; G2 and G3 are retained from the prior version. SFA weather rows are duplicated in Panel B to allow one-to-one comparison with FIML's abatement-equation weather.

Figure 1: Multi-ware Efficiency and Regime Interactions in the Land-Pollution Set



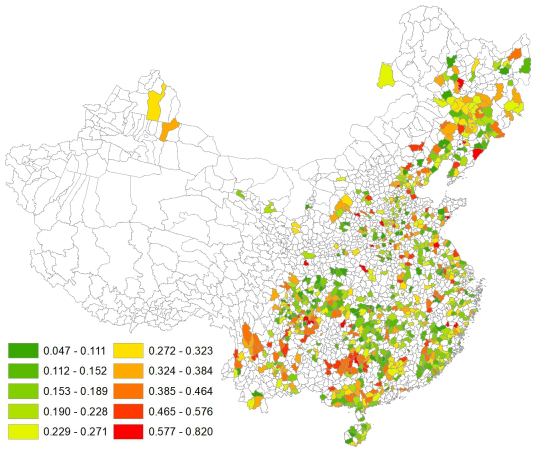
(a) Separable frontiers



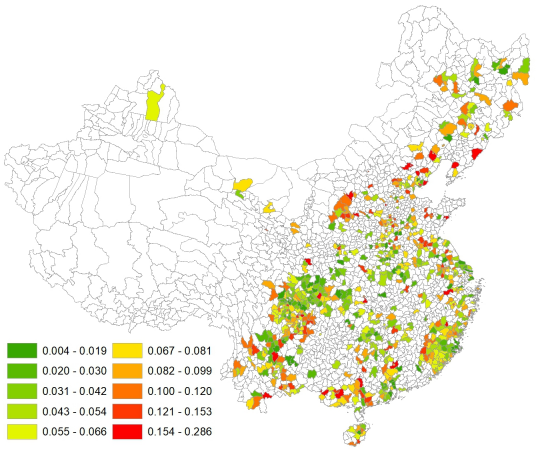
(b) The "kink" with multi-hyperplane tangency

Notes: Both panels are drawn in the pollution–land (b, l) netput space, holding other inputs $\bar{x}$, outputs $\bar{y}$, and abatement services $\bar{e}$ fixed. **In panel (a)**, two efficiency frontiers are shown: the production frontier σ_y (black) and the abatement frontier σ_a (blue). Their intersection defines the feasible upper contour set of the multi-ware technology, whose boundary σ_Z (orange) represents the joint efficient frontier. The interaction zone is partitioned into locally dominant regimes: the production regime, where σ_y binds, and the abatement regime, where σ_a binds. **In panel (b)**, the meeting point of these regimes is marked as a kinked boundary, characterized by multiple supporting hyperplanes representing distinct local shadow prices on either side of the intersection.

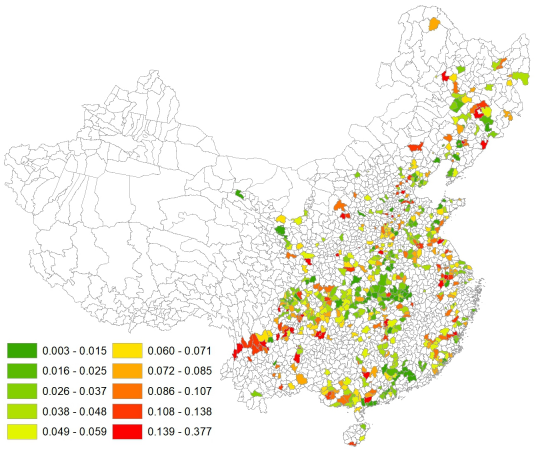
Figure 2: Shadow Values of Abated NH_3-N and Marginal Product of Pollution Abated (MPPA) by Farm Group in Mainland China under the FIML Approach



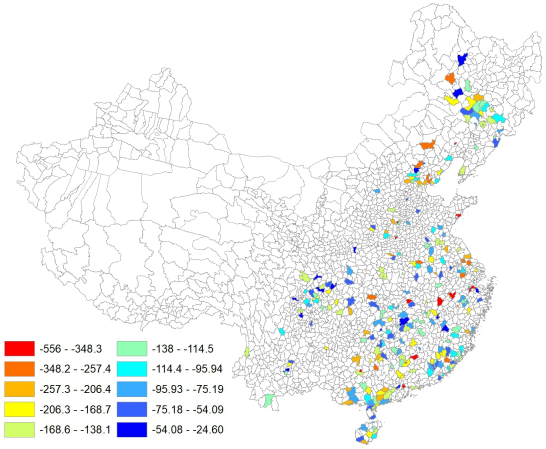
(a) Shadow Value, Group 1 *(mean = 0.291)*



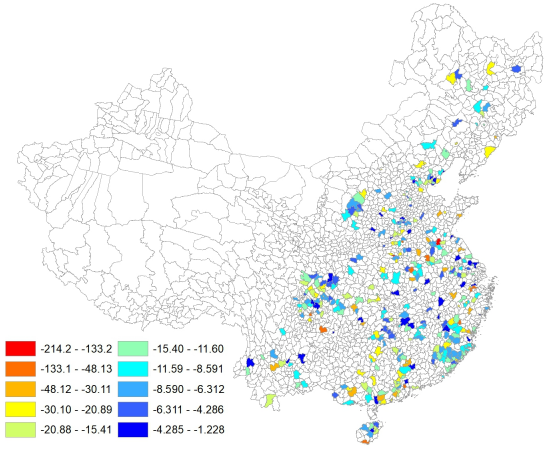
(b) Shadow Value, Group 2 *(mean = 0.073)*



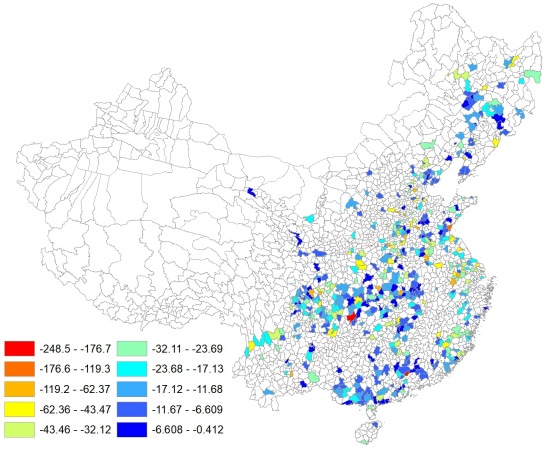
(c) Shadow Value, Group 3 (*mean = 0.059)*



(d) MPPA, Group 1 (*mean = -163.34)*



(e) MPPA, Group 2 (*mean = -9.29)*



(f) MPPA, Group 3 (*mean = -13.92)*

Notes. Each panel maps county-level estimates for mainland China. Panels (a)-(c) show shadow values of abated NH_3-N, measured in land area (unit: 1000 m^2 $\approx$ 0.247 acres). Panels (d)-(f) show the marginal product of pollution abated (MPPA), defined as $-SP_{y_a}/SP_y$, where SP_{y_a} and SP_y denote the shadow prices of abated NH_3-N and meat output, respectively.

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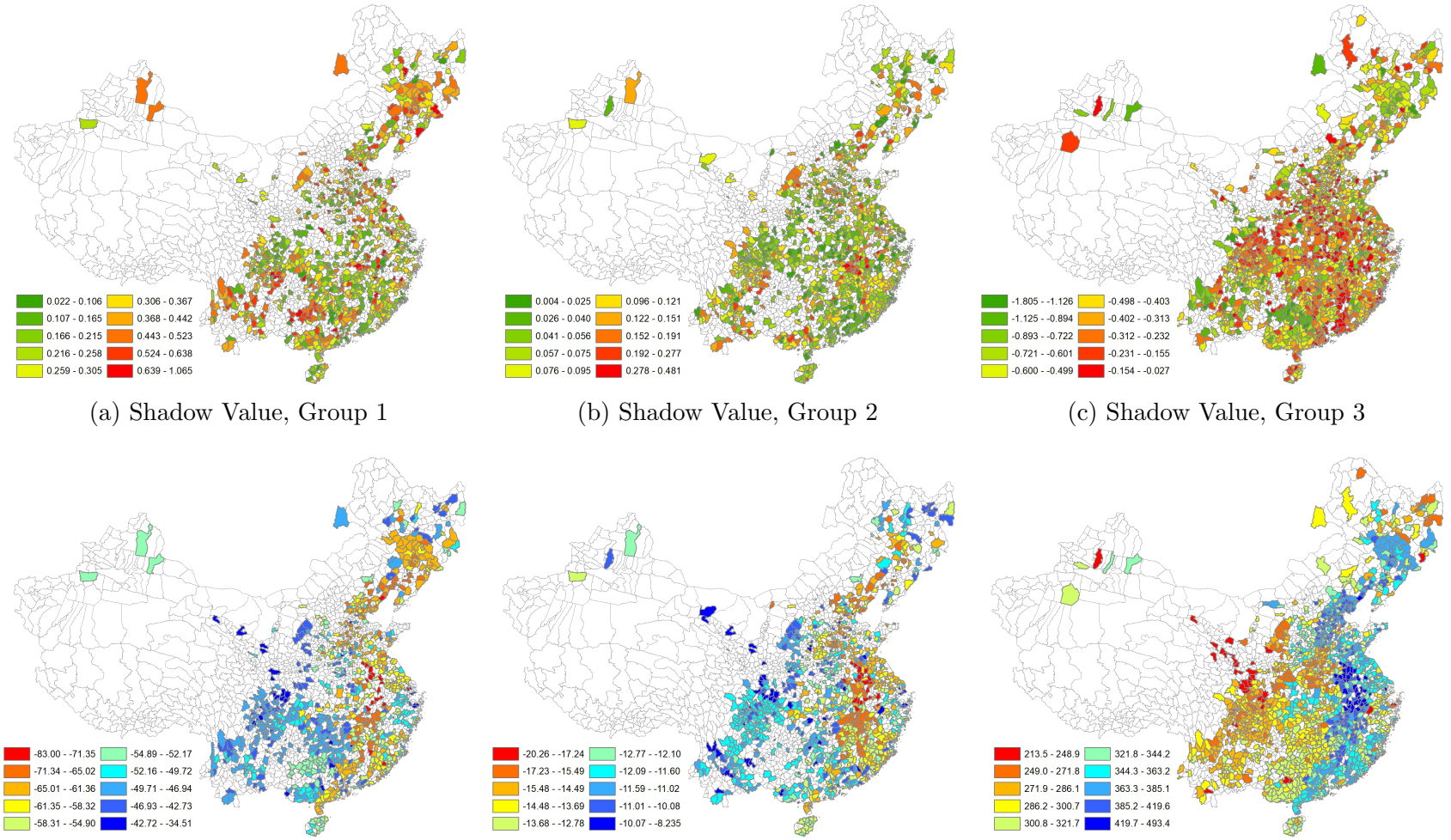
Figure A1: Shadow Values of Abated NH_3-N and Marginal Product of Pollution Abated (MPPA) by Farm Group in Mainland China under the Single-Frontier Approach



(a) Shadow Value, Group 1 (b) Shadow Value, Group 2 (c) Shadow Value, Group 3

(d) MPPA, Group 1 (e) MPPA, Group 2 (f) MPPA, Group 3

Notes. Each panel maps county-level estimates for mainland China. Panels (a)-(c) show shadow values of abated NH_3-N, measured in land area (unit: 1000 m^2). Panels (d)-(f) show the marginal product of pollution abated (MPPA), defined as $-SP_{y_a}/SP_y$, where SP_{y_a} and SP_y denote the shadow prices of abated NH_3-N and meat output, respectively.